Exercise 1 (deadline: October 9th, 4pm)

Exercise 3.1

The following formula has its parentheses removed. Restore the parentheses.

$$\neg p_1 \rightarrow \neg\neg p_2 \leftrightarrow p_3 \land p_4 .$$

Exercise 3.9

Show that the formulas $p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$ are not equivalent by finding an interpretation in which they have different truth values.

Exercise 2 (deadline: October 16th, 4pm)

Exercise 4.2 (b)
Build a truth table for the following formula

$$p \leftrightarrow (\neg r \rightarrow \neg p).$$

Exercise 4.5
Check, using splitting, whether the formula
$(p \leftrightarrow q) \wedge ((p \wedge \neg q) \vee (q \wedge \neg p))$ is satisfiable. Split on the atom p first.

Exercise 5.4 (a)
Apply the standard CNF transformation algorithm to the following formula:

$$\neg((p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p))$$

Exercise 3 (deadline: October 22nd, 4pm)

Exercise 4.16 (b)

Draw the parse tree for the formula $r \land \neg p \land q \rightarrow ((p \leftrightarrow \neg q) \rightarrow r)$ and mark the nodes corresponding to the negative occurrences of subformulas (e.g., encircle them). Write down all negative subformulas of this formula.

Part of Exercise 5.4

Apply the definitional clausal form transformation algorithm (the non-optimized version) to the formula

$$\neg((p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p))$$

Exercise 5.7

Apply the DPLL algorithm to the following sets of clauses:

$$\begin{array}{llll} p \lor q \lor r, & \neg p \lor \neg q \lor \neg r, & p \lor \neg q \lor \neg r, & \neg p \lor q, \\ \neg p \lor r, & p \lor q \lor \neg r, & p \lor \neg q \lor r. & \end{array}$$

Is this set satisfiable? If yes, find a model of this set.

Exercise 5.8

Apply the DPLL algorithm to the following sets of clauses:

$$\begin{array}{lll} p_1 \lor p_3, & \neg p_2 \lor \neg p_3, & p_1 \lor \neg p_3, \\ \neg p_1 \lor p_2, & p_1 \lor p_2 \lor \neg p_3, & p_1 \lor p_2 \lor p_3. \end{array}$$

Is this set satisfiable? If yes, find a model of this set.

Exercise 4 (deadline: October 29th, 4pm)

Part of Exercise 8.8

Consider the set consisting of the following clauses:

$$\neg p_0 \lor \neg p_1 \lor \neg p_2, \quad p_0 \lor \neg p_2, \quad \neg p_0 \lor p_1, \quad p_1 \lor p_2, \quad \neg p_0 \lor \neg p_1 \lor p_2.$$

Show how GSAT can find a model of this set starting with the initial random interpretation $\{p_0 \mapsto 1, p_1 \mapsto 0, p_2 \mapsto 1\}$.

Exercise 5 (deadline: November 5th, 4pm)

Part of Exercise 8.8

Consider the set consisting of the following clauses:

$$\neg p_0 \lor \neg p_1 \lor \neg p_2, \quad p_0 \lor \neg p_2, \quad \neg p_0 \lor p_1, \quad p_1 \lor p_2, \quad \neg p_0 \lor \neg p_1 \lor p_2.$$

Show how WSAT can find a model of this set starting with the initial random interpretation $\{p_0 \mapsto 1, p_1 \mapsto 0, p_2 \mapsto 1\}$.

Part of Exercise 8.10

Consider the set consisting of the following clauses:

$p_0 \lor \neg p_1 \lor p_2$	$p_0 \lor \neg p_1 \lor p_2 \lor p_4$	$\neg p_0 \lor p_1 \lor \neg p_2$	$\neg p_0 \lor \neg p_1 \lor \neg p_2 \lor \neg p_4$
$p_0 \lor \neg p_1 \lor p_4$	$p_3 \lor p_2 \lor p_4 \lor \neg p_0$	$\neg p_2 \lor \neg p_2 \lor p_4 \lor p_3$	$\neg p_2 \lor \neg p_0 \lor p_4 \lor p_4$
$p_0 \lor p_3 \lor \neg p_4$	$p_0 \lor \neg p_1 \lor \neg p_2 \lor \neg p_3$	$\neg p_1 \lor \neg p_2 \lor \neg p_3$	$p_1 \lor \neg p_2 \lor \neg p_3 \lor \neg p_4$
$p_1 \lor p_2$	$p_2 \lor p_3 \lor \neg p_4 \lor p_3$	$\neg p_0 \lor \neg p_2 \lor \neg p_3 \lor \neg p_4$	$p_0 \lor p_2 \lor p_4$

For each of the variables p_0, p_1, p_2, p_2, p_4 find the probability that WSAT will choose this variable for flipping when the current interpretation is $\{p_0 \mapsto 0, p_1 \mapsto 0, p_2 \mapsto 0, p_3 \mapsto 0, p_4 \mapsto 0\}$.

Part of Exercise 9.3

Show validity of the following formula using semantic tableaux:

$$(p \to r) \to (p \lor q \to r \lor q).$$

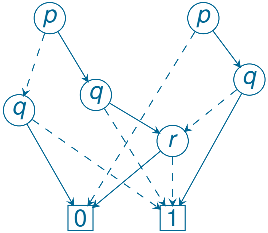
Exercise 6 (deadline: November 19th, 4pm)

Part of Exercise 10.1

Compute the OBDD for the formula $\neg(p_1 \wedge p_2) \rightarrow (p_1 \vee p_3)$ and the order $p_3 > p_2 > p_1$.

Exercise 10.3

Consider the following global dag D.



It has two different subdags d_1, d_2 rooted at p. Let d_1, d_2 represent formulas F_1, F_2, respectively. Draw the global dag D after the OBDD for $F_1 \wedge F_2$ has been integrated into it.

Exercise 10.5

A propositional formula F of variables $p_1, \ldots, p_n$ is true in an interpretation I if and only if exactly one atom from $p_1, \ldots, p_n$ is true in I. Draw the OBDD for F and the order $p_1 > p_2 > \ldots$.

Exercise 7 (deadline: November 26th, 4pm)

Built from Exercises 11.2 and 11.3

Draw the parse tree for the following formula:

$$\forall p(p \rightarrow \exists q(q \rightarrow \forall r(p \vee q \vee r))).$$

Underline all free occurrences of variables in this formula.

Part of Exercise 11.6

Transform the following formula into prenex form:

$$\forall p\, \neg p \vee \forall p\, p \rightarrow \neg p.$$

Part of Exercise 11.9

Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p(p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)).$$

Exercise 8 (deadline: December 3rd, 4pm)

Part of Exercise 11.15

Evaluate the following formula using only the pure literal rule, universal literal deletion and unit propagation.

$$\forall p \exists q \forall s \exists r ((p \vee q \vee s) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee r \vee \neg s)).$$

Part of Exercise 11.17

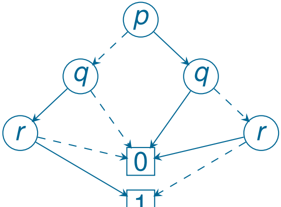
Evaluate the following formula using DPLL:

$$\forall p \exists q \forall s \exists r ((p \vee q \vee s) \wedge (p \vee \neg q \vee \neg r \vee \neg s) \wedge (p \vee \neg q \vee r \vee \neg s))$$

Part of Exercise 11.20

A formula F has the OBDD shown on the right.

Apply the quantification algorithm to this OBDD to obtain the OBDD for the formula $\exists q F$.



Exercise 9 (deadline: December 10th, 4pm)

Exercise 12.13

Let x be a variable with the domain $\{u, v, w\}$ and p be a boolean variable. Transform the following formula of PLFD into a propositional formula: $\neg x = v \rightarrow x = u \wedge p = 0$.

The following two exercises use the vending machine system from the lectures and the reading material.

Exercise 13.1 (part 2)

We know that students only drink beer while professors only drink coffee. Represent the set of states in which there is a drink in the dispenser but it does not suit the current customer.

Exercise 13.2 (part 1)

Represent symbolically the money-swallowing transition: this transition can remove any amount of coins from the coin slot.

Exercise 13.9

The variable x range over the domain $\{1, 2, 3\}$. Represent the transition which strictly increases the value of x.

Exercise 10 (deadline: December 17th, 4pm)

Exercise 14.2 (part 1)

Express in LTL the following properties: if F occurs at least twice, then F occurs infinitely often.

Exercise 14.5 (part 3)

Show that the formulas $\square(A \rightarrow \bigcirc A)$ and $\neg A\,\mathcal{U}\,\square A$ are not equivalent by giving a path that satisfies one of them but does not satisfy the other one.

Exercise (not in the reading material)

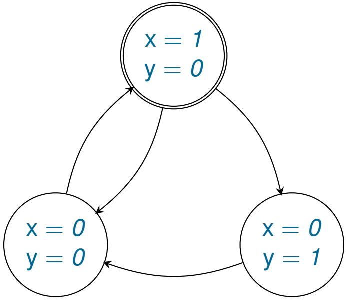
Let the formula x symbolically represent the set of initial states and the formula $x \leftrightarrow \neg x'$ the transition relation of a transition system $\mathbb{S}$ over the set of two boolean variables $\{x, y\}$.

- Draw the state transition graph of $\mathbb{S}$.
- Which of the following formulas are true along all paths in $\mathbb{S}$?
  1. $\square(x \leftrightarrow \bigcirc\neg x)$;
  2. $\square(x \leftrightarrow \bigcirc\bigcirc\neg x)$;
  3. $\square(y \leftrightarrow \bigcirc y)$.

Outline

Exercise 11. Problem 1

Consider a transition system with the following state transition graph.



The initial state is the top one. Let S_1 be the set of states symbolically represented by the formula $x = 1$ and S_2 be the set of states symbolically represented by the formula $x = 0 \land y = 1$.

1. State whether or not S_1 coincide with the set of initial states.
2. Find a symbolic representation of the set of states reachable from S_2 in exactly two steps.
3. Find a symbolic representation of the set of states backward reachable from S_2 in exactly three steps.