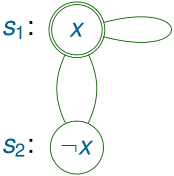
# Outline

# Computation Tree

Let $\mathbb{S} = (S, In, T, \mathcal{X}, dom, L)$ be a transition system and $s \in S$ be a state. The computation tree for $\mathbb{S}$ starting at $s$ is the following (possibly infinite) tree.

1. The nodes of the tree are labeled by states in $S$.
2. The root of the tree is labeled by $s$.
3. For every node $s'$ in the tree, its children are exactly such nodes $s'' \in S$ that $(s', s'') \in T$.

# Computation Trees and Paths



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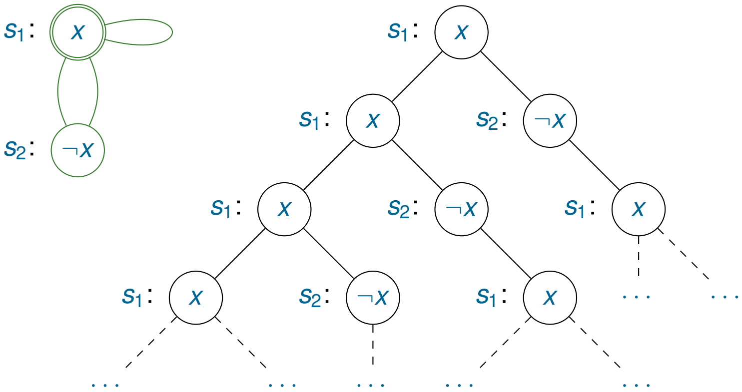

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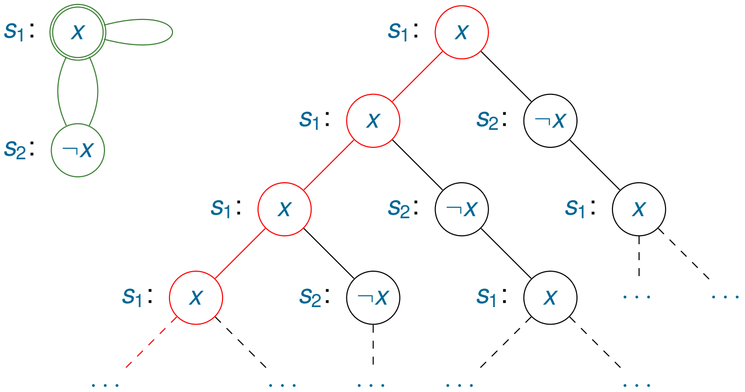


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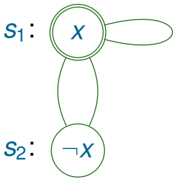


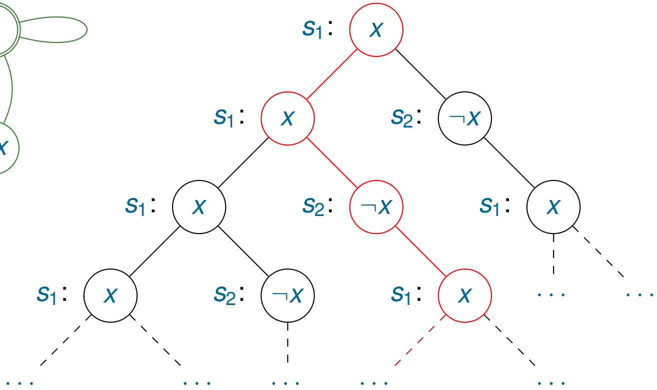


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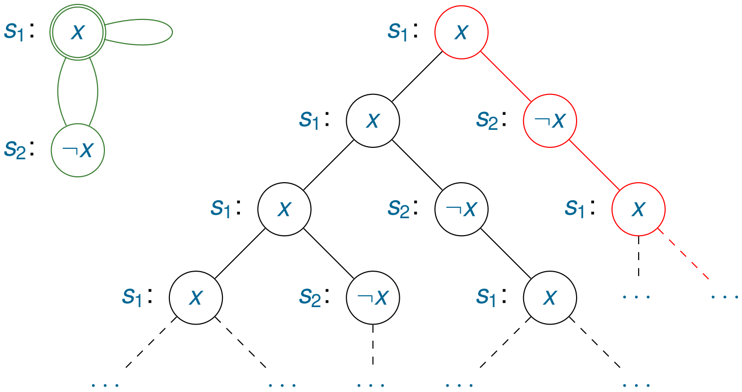


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# Computation

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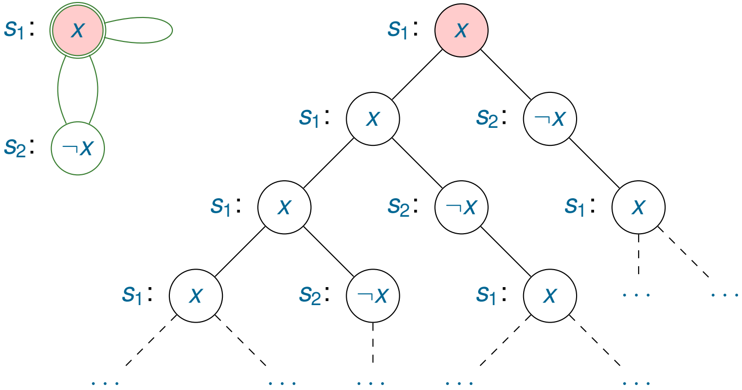

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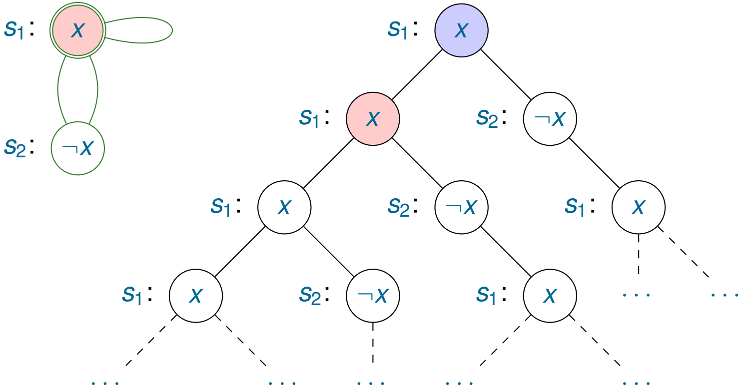

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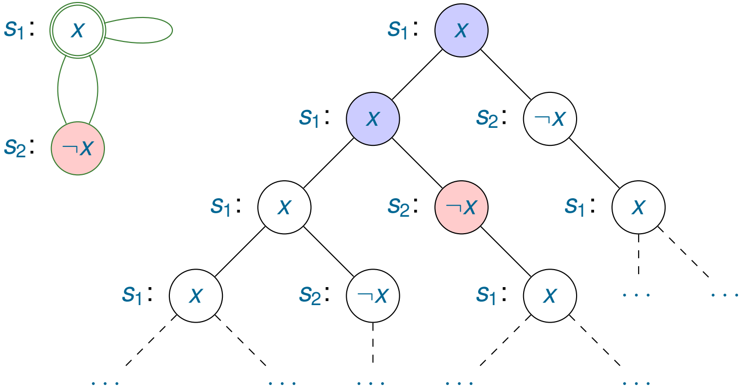

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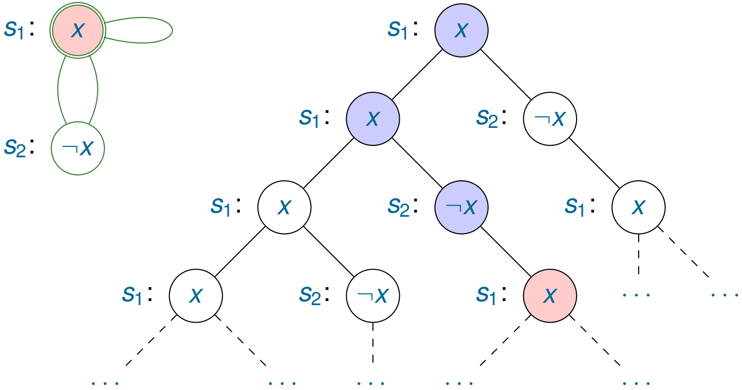

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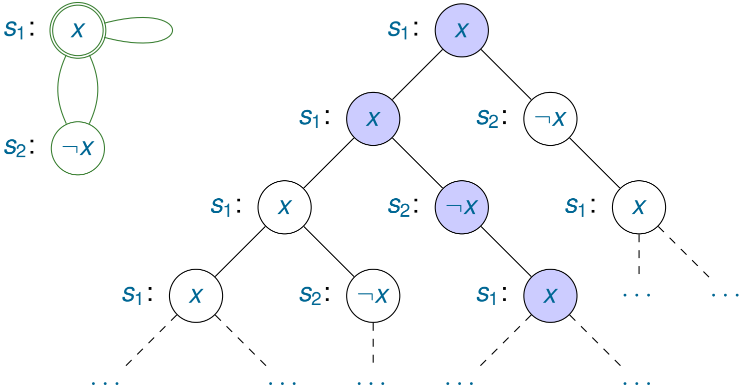

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- For every transition system $\mathbb{S}$ and state $s$ there exists a unique computation tree for $\mathbb{S}$ starting at $s$, up to the order of children.

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1. If $F$ is a formula, then $\bigcirc F$, $\square F$, and $\lozenge F$ are formulas;
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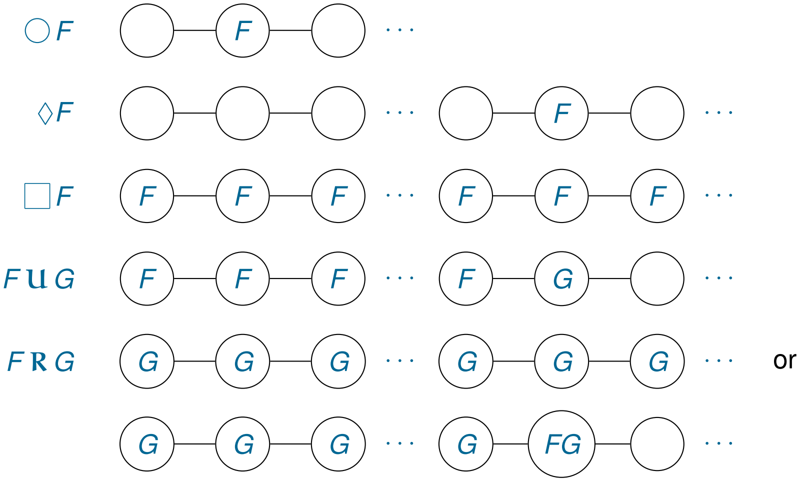
| | |
|---|---|
| $\bigcirc$ | next |
| $\square$ | always (in future) |
| $\lozenge$ | sometimes (in future) |
| $\mathcal{U}$ | until |
| $\mathcal{R}$ | release |

# Semantics (intuitive)



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Let $\pi = s_0, s_1, s_2 \ldots$ be a sequence of states and $F$ be an $\mathrm{LTL}$ formula.

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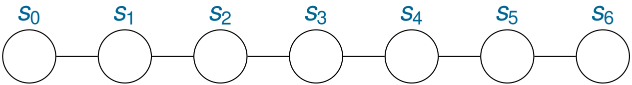

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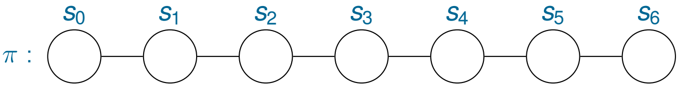


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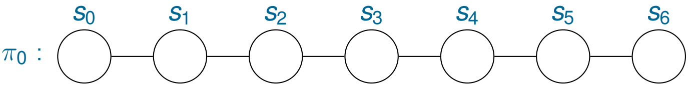


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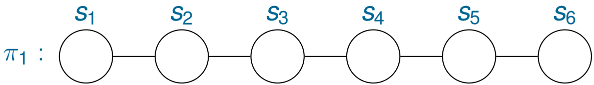


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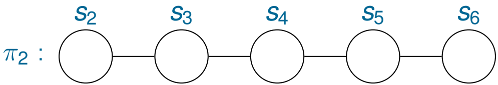


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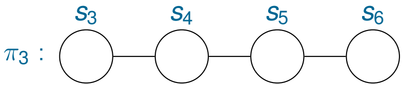


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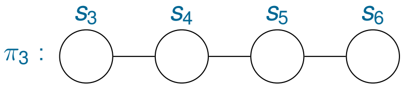


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For all $i = 0, 1, \ldots$ denote by $\pi_i$ the sequence of states $s_i, s_{i+1}, s_{i+2} \ldots$ (note that $\pi_0 = \pi$).
To define $\pi \models F$ we will use $\pi_i \models G$ for some $i$ and $G$. We will sometimes (slightly informally) say that $G$ is true in $s_i$ or $G$ holds in $s_i$ to mean that $G$ is true on $\pi_i$.

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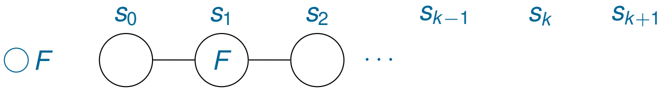
3. $\pi \models F_1 \land \ldots \land F_n$ if for all $j = 1, \ldots, n$ we have $\pi \models F_j$;
   $\pi \models F_1 \lor \ldots \lor F_n$ if for some $j = 1, \ldots, n$ we have $\pi \models F_j$.
4. $\pi \models \neg F$ if $\pi \not\models F$.
5. $\pi \models F \to G$ if either $\pi \not\models F$ or $\pi \models G$;
   $\pi \models F \leftrightarrow G$ if either both $\pi \not\models F$ and $\pi \not\models G$ or both $\pi \models F$ and $\pi \models G$.

# Semantics of temporal operators

6. $\pi \models \bigcirc F$ if $\pi_1 \models F$;



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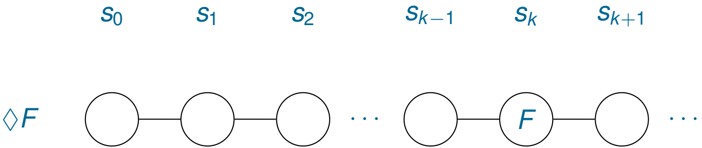

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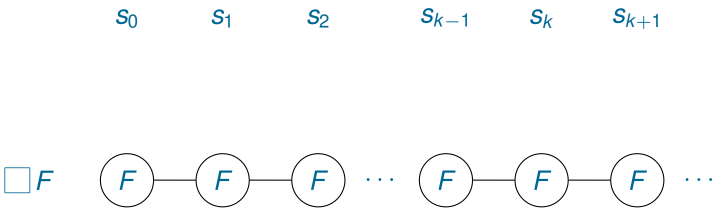

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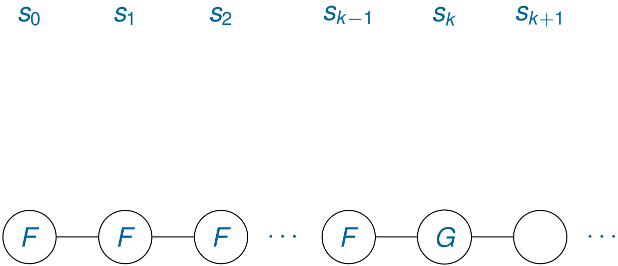

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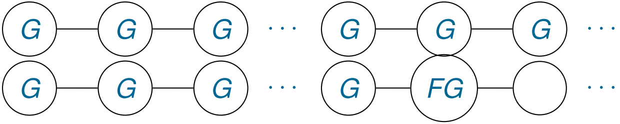
$s_0$ $s_1$ $s_2$ $s_{k-1}$ $s_k$ $s_{k+1}$

$F \,\mathcal{R}\, G$

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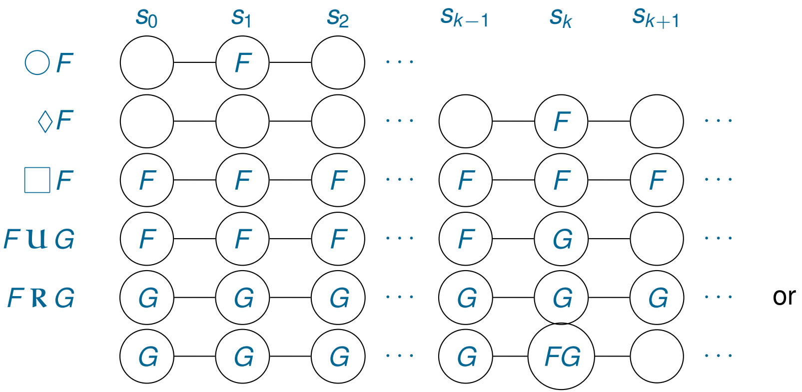

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For an $\mathrm{LTL}$ formula $F$ we can consider two kinds of properties of $\mathbb{S}$:

1. does $F$ hold on some computation path for $\mathbb{S}$ from an initial state?
2. does $F$ hold on all computation paths for $\mathbb{S}$ from an initial state?

# Precedences of Connectives and Temporal Operators

| Connective | Precedence |
|---|---|
| $\neg, \bigcirc, \Diamond, \Box$ | 5 |
| $\mathbf{U}, \mathbf{R}$ | 4 |
| $\wedge, \vee$ | 3 |
| $\rightarrow$ | 2 |
| $\leftrightarrow$ | 1 |

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4. $F$ holds in at least two states. $\Diamond(F \wedge \bigcirc\Diamond F)$
5. Unless $s_i$ is the first state of the path, if $F$ holds in state $s_i$, then $G$ must hold in at least one of the two states just before $s_i$, that is $s_{i-1}$ and $s_{i-2}$.

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3. $F$ holds in at most one state. $\square(F \rightarrow \bigcirc\square\neg F)$
4. $F$ holds in at least two states. $\Diamond(F \wedge \bigcirc\Diamond F)$
5. Unless $s_i$ is the first state of the path, if $F$ holds in state $s_i$, then $G$ must hold in at least one of the two states just before $s_i$, that is $s_{i-1}$ and $s_{i-2}$. $(\bigcirc F \rightarrow G) \wedge \square(\bigcirc\bigcirc F \rightarrow G \vee \bigcirc G)$
6. $F$ happens infinitely often. $\square\Diamond F$
7. $F$ holds in each even state and does not hold in each odd state (states are counted from $0$).

# Expressing Some Properties

1. $F$ never holds in two consecutive states. $\Box(F \to \bigcirc\neg F)$
2. If $F$ holds in a state $s$, it also holds in all states after $s$.
   $\Box(F \to \Box F)$
3. $F$ holds in at most one state. $\Box(F \to \bigcirc\Box\neg F)$
4. $F$ holds in at least two states. $\Diamond(F \land \bigcirc\Diamond F)$
5. Unless $s_i$ is the first state of the path, if $F$ holds in state $s_i$, then $G$ must hold in at least one of the two states just before $s_i$, that is $s_{i-1}$ and $s_{i-2}$. $(\bigcirc F \to G) \land \Box(\bigcirc\bigcirc F \to G \lor \bigcirc G)$
6. $F$ happens infinitely often. $\Box\Diamond F$
7. $F$ holds in each even state and does not hold in each odd state (states are counted from $0$). $F \land \Box(F \leftrightarrow \bigcirc\neg F)$.

# Not all “reasonable” properties are expressible in LTL

$p$ holds in all even states.

# End of Lecture 19

Slides for lecture 19 end here . . .

# Meaning of Some Formulas

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# Meaning of Some Formulas

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- $\Box \Diamond F$;
- $F \land \Box(F \leftrightarrow \neg \bigcirc F)$;

# Equivalences: Unwinding Properties

$$
\begin{array}{rcl}
\Diamond F & \equiv & F \vee \bigcirc \Diamond F \\
\Box F & \equiv & F \wedge \bigcirc \Box F \\
F \,\mathcal{U}\, G & \equiv & G \vee (F \wedge \bigcirc (F \,\mathcal{U}\, G)) \\
F \,\mathcal{R}\, G & \equiv & G \wedge (F \vee \bigcirc (F \,\mathcal{R}\, G))
\end{array}
$$

# Equivalences: Negation of Temporal Operators

$$
\begin{array}{rcl}
\neg \bigcirc F & \equiv & \bigcirc \neg F \\
\neg \Diamond F & \equiv & \Box \neg F \\
\neg \Box F & \equiv & \Diamond \neg F \\
\neg (F \,\mathbf{U}\, G) & \equiv & \neg F \,\mathbf{R}\, \neg G \\
\neg (F \,\mathbf{R}\, G) & \equiv & \neg F \,\mathbf{U}\, \neg G
\end{array}
$$

# Expressing Temporal Operators Using $\mathcal{U}$

$$
\begin{array}{rcl}
\Diamond F & \equiv & \top \,\mathcal{U}\, F \\
\Box F & \equiv & \neg(\top \,\mathcal{U}\, \neg F) \\
F \,\mathcal{R}\, G & \equiv & \neg(\neg F \,\mathcal{U}\, \neg G).
\end{array}
$$

Therefore, all operators can be expressed using $\bigcirc$ and $\mathcal{U}$.

# Other Equivalences

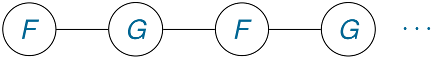
$$
\begin{aligned}
\Diamond(F \vee G) &\equiv \Diamond F \vee \Diamond G \\
\Box(F \wedge G) &\equiv \Box F \wedge \Box G
\end{aligned}
$$

But

$$
\begin{aligned}
\Box(F \vee G) &\not\equiv \Box F \vee \Box G \\
\Diamond(F \wedge G) &\not\equiv \Diamond F \wedge \Diamond G
\end{aligned}
$$

# How to Show that Two Formulas are not Equivalent?

Find a path that satisfies one of the formulas but not the other. For example for $\square(F \vee G)$ and $\square F \vee \square G$.



# Formalization: Variables and Domains

| variable | domain | explanation |
|---|---|---|
| st_coffee | $\{0, 1\}$ | drink storage contains coffee |
| st_beer | $\{0, 1\}$ | drink storage contains beer |
| disp | $\{none, beer, coffee\}$ | content of drink dispenser |
| coins | $\{0, 1, 2, 3\}$ | number of coins in the slot |
| customer | $\{none, student, prof\}$ | customer |

# Transitions

1. *Recharge* which results in the drink storage having both beer and coffee.
2. *Customer_arrives*, after which a customer appears at the machine.
3. *Customer_leaves*, after which the customer leaves.
4. *Coin_insert*, when the customer inserts a coin in the machine.
5. *Dispense_beer*, when the customer presses the button to get a can of beer.
6. *Dispense_coffee*, when the customer presses the button to get a cup of coffee.
7. *Take_drink*, when the customer removes a drink from the dispenser.

# Reasoning About Transitions

Consider the following properties:

1. "one cannot have two beers in a row without inserting a coin".
2. "If we never have two recharge transitions in a row, then the next transition after a recharge must be a customer arrival".

Note that they are about transitions, not about states.

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How can one represent these properties?

Introduce a state variable denoting the next transition.

# Example

$$\begin{aligned}
\mathit{Recharge} &\stackrel{\text{def}}{=} \mathrm{tr} = \mathit{Recharge} \wedge \mathrm{customer} = \mathit{none} \wedge \\
&\quad \mathrm{st_coffee}' \wedge \mathrm{st_beer}' \wedge \\
&\quad \mathit{only}(\mathrm{st_coffee}, \mathrm{st_beer}, \mathrm{tr}). \\
\mathit{Customer_arrives} &\stackrel{\text{def}}{=} \mathrm{tr} = \mathit{Customer_arrives} \wedge \mathrm{customer} = \mathit{none} \wedge \\
&\quad \mathrm{customer}' \neq \mathit{none} \wedge \\
&\quad \mathit{only}(\mathrm{customer}, \mathrm{tr}) \\
\mathit{Coin_insert} &\stackrel{\text{def}}{=} \mathrm{tr} = \mathit{Coin_insert} \wedge \\
&\quad \mathrm{customer} \neq \mathit{none} \wedge \mathrm{coins} \neq \mathit{3} \wedge \\
&\quad (\mathrm{coins} = \mathit{0} \rightarrow \mathrm{coins}' = \mathit{1}) \wedge \\
&\quad (\mathrm{coins} = \mathit{1} \rightarrow \mathrm{coins}' = \mathit{2}) \wedge \\
&\quad (\mathrm{coins} = \mathit{2} \rightarrow \mathrm{coins}' = \mathit{3}) \wedge \\
&\quad \mathit{only}(\mathrm{coins}, \mathrm{tr}).
\end{aligned}$$

# Representing Temporal Properties of Transitions

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$$\Box(\bigwedge_{v \in \textit{dom}(\text{customer})}(\text{customer} = v \wedge \bigcirc\text{customer} \neq v) \rightarrow \\ \text{tr} = \textit{Customer\_arrives} \vee \text{tr} = \textit{Customer\_leaves})$$

# Representing Temporal Properties of Transitions

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$$\Box\Diamond \text{tr} = \mathit{Recharge} \rightarrow \\ \Box(\text{tr} = \mathit{Dispense\_beer} \rightarrow \Diamond \text{tr} = \mathit{Customer\_leaves})$$

# End of Lecture 20

Slides for lecture 20 end here . . .