

# Outline

Semantic Tableaux

# Signed Formula

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- ▶ If  $A = b$  is true in  $I$ , we also say that  $I$  **is a model of**  $A = b$ , or that  $I$  **satisfies**  $A = b$ .
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Note:

1. For every formula  $A$  and interpretation  $I$  **exactly one** of the signed formulas  $A = 1$  and  $A = 0$  is true in  $I$ .
2. A formula  $A$  is **satisfiable** if and only if so is the signed formula  $A = 1$ .

# How to find a model of a signed formula?

Example:  $(A \rightarrow B) = 1$ .

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Operation table for  $\rightarrow$ :

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

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Likewise,  $(A \rightarrow B) = 0$  if and only  
if  $A = 1$  AND  $B = 0$ .

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Likewise,  $(A \rightarrow B) = 0$  if and only  
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So we can use **AND-OR** trees to  
carry out case analysis.

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
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# Tableau

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**Tableau for a signed formula**  $A = b$  has  $A = b$  as a root.

Alternatively, we can regard a tableau as a collection of **branches**; each branch is a set of signed formulas.

**Notation for a set of branches**:  $B_1 \mid \dots \mid B_n$ , where each of the  $B_i$  is a branch.

# Branch Expansion Rules

$$(A_1 \wedge \dots \wedge A_n) = 0 \rightsquigarrow A_1 = 0 \mid \dots \mid A_n = 0$$

$$(A_1 \wedge \dots \wedge A_n) = 1 \rightsquigarrow A_1 = 1, \dots, A_n = 1$$

$$(A_1 \vee \dots \vee A_n) = 0 \rightsquigarrow A_1 = 0, \dots, A_n = 0$$

$$(A_1 \vee \dots \vee A_n) = 1 \rightsquigarrow A_1 = 1 \mid \dots \mid A_n = 1$$

$$(A_1 \rightarrow A_2) = 0 \rightsquigarrow A_1 = 1, A_2 = 0$$

$$(A_1 \rightarrow A_2) = 1 \rightsquigarrow A_1 = 0 \mid A_2 = 1$$

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- ▶ it contains both  $p = 0$  and  $p = 1$  for some atom  $p$
- ▶ it contains  $\top = 0$ ;
- ▶ it contains  $\perp = 1$ .

# A Semantic Tableau

$$(\neg(q \vee p \rightarrow p \vee q)) = 1$$

$$\begin{aligned}(A_1 \vee A_2) = 0 &\rightsquigarrow A_1 = 0, A_2 = 0 \\(A_1 \vee A_2) = 1 &\rightsquigarrow A_1 = 1 \mid A_2 = 1 \\(A_1 \rightarrow A_2) = 0 &\rightsquigarrow A_1 = 1, A_2 = 0 \\(\neg A_1) = 1 &\rightsquigarrow A_1 = 0\end{aligned}$$

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$$\begin{array}{l} p = 0 \\ q = 1 \end{array}$$

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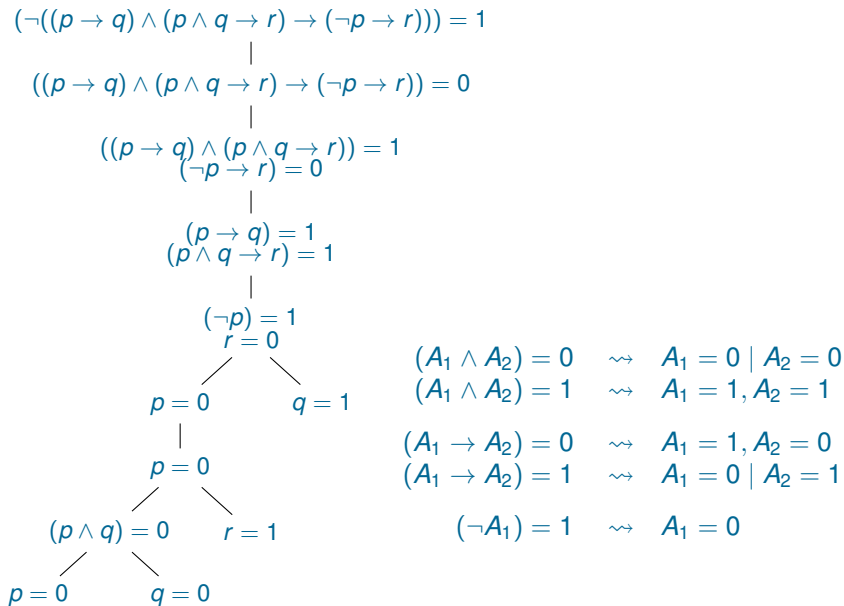
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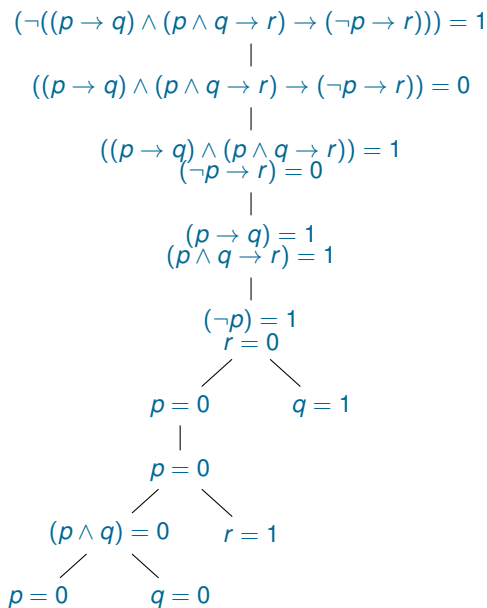
$$(A_1 \rightarrow A_2) = 1 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 1$$

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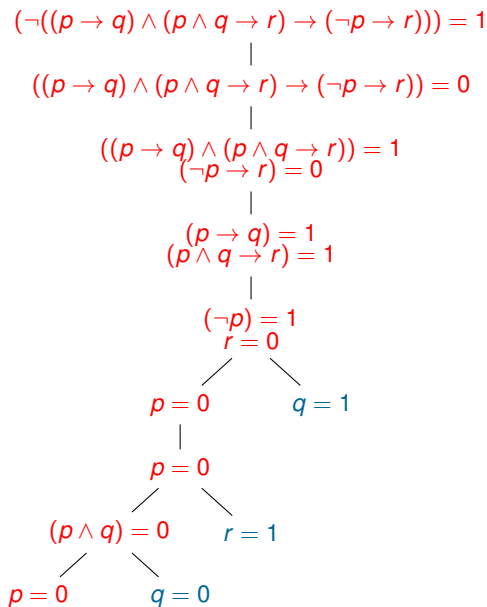
All rules on this branch have been applied, so the formula is **satisfiable**.



# Finding Models Using Tableaux

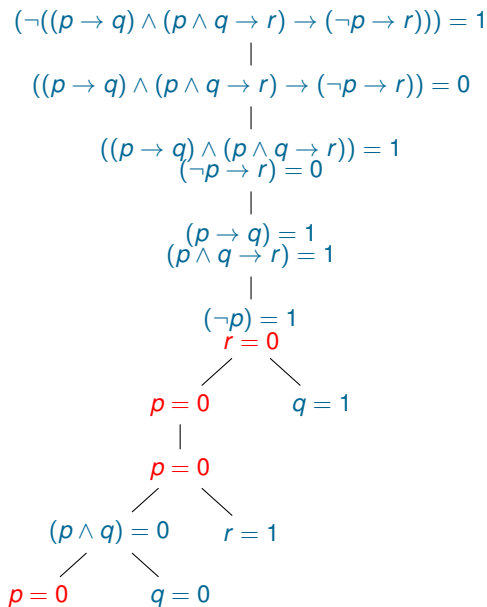


# Finding Models Using Tableaux



Build an open branch on which all rules have been applied.

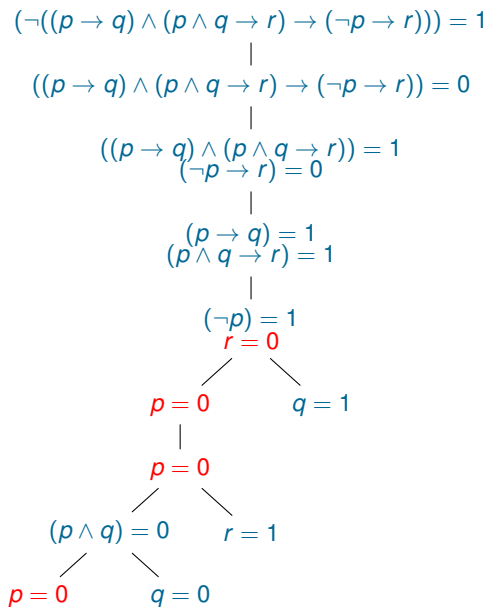
# Finding Models Using Tableaux



Build an open branch on which all rules have been applied.

Select signed atoms on this branch

# Finding Models Using Tableaux



Build an open branch on which all rules have been applied.

Select signed atoms on this branch

They give us a model

$$\{r \mapsto 0, p \mapsto 0, q \mapsto \dots\}$$

# Checking Other Properties with Tableaux

A formula  $A$  is **satisfiable** iff a tableau for  $A = 1$  contains a complete open branch (and iff every tableau for  $A = 1$  contains a complete open branch).

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A **fully expanded tableau** for  $A = 1$  gives us **all models** of  $A$ .



# Alternative View of Tableaux

We will make the following changes:

1. show a tableau using the  $B_1 \mid \dots \mid B_n$  notation;
2. remove closed branches;
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All branches are closed, so the signed formula  $(\neg(q \vee p \rightarrow p \vee q)) = 1$  is unsatisfiable.

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$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p \rightarrow r) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, p = 0, r = 0 \rightsquigarrow$$

$$(p \rightarrow q) = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0$$

## Alternative View of Tableaux: Example 2

$$(\neg((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r))) = 1 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r)) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p \rightarrow r) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, p = 0, r = 0 \rightsquigarrow$$

$$(p \rightarrow q) = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0$$

## Alternative View of Tableaux: Example 2

$$(\neg((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r))) = 1 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r)) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p \rightarrow r) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, p = 0, r = 0 \rightsquigarrow$$

$$(p \rightarrow q) = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$$

$$p = 0, (p \wedge q \rightarrow r) = 1, r = 0 \mid$$

$$q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0$$

## Alternative View of Tableaux: Example 2

$$(\neg((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r))) = 1 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r)) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p \rightarrow r) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, p = 0, r = 0 \rightsquigarrow$$

$$(p \rightarrow q) = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$$

$$p = 0, (p \wedge q \rightarrow r) = 1, r = 0 \mid$$

$$q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0$$

## Alternative View of Tableaux: Example 2

$$(\neg((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r))) = 1 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r)) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p \rightarrow r) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, p = 0, r = 0 \rightsquigarrow$$

$$(p \rightarrow q) = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$$

$$p = 0, (p \wedge q \rightarrow r) = 1, r = 0 \mid$$

$$q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$$

$$p = 0, (p \wedge q) = 0, r = 0 \mid$$

$$p = 0, r = 1, r = 0 \mid$$

$$q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0$$

## Alternative View of Tableaux: Example 2

$$(\neg((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r))) = 1 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r)) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p \rightarrow r) = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, p = 0, r = 0 \rightsquigarrow$$

$$(p \rightarrow q) = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$$

$$p = 0, (p \wedge q \rightarrow r) = 1, r = 0 \mid$$

$$q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$$

$$p = 0, (p \wedge q) = 0, r = 0 \mid$$

$$p = 0, r = 1, r = 0 \mid$$

$$q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0$$

## Alternative View of Tableaux: Example 2

$(\neg((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r))) = 1 \rightsquigarrow$   
 $((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r)) = 0 \rightsquigarrow$   
 $((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p \rightarrow r) = 0 \rightsquigarrow$   
 $((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$   
 $((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, p = 0, r = 0 \rightsquigarrow$   
 $(p \rightarrow q) = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$   
 $p = 0, (p \wedge q \rightarrow r) = 1, r = 0 \mid$   
 $q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$   
 $p = 0, (p \wedge q) = 0, r = 0 \mid$   
 $p = 0, r = 1, r = 0 \mid$   
 $q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$   
 $p = 0, r = 0 \mid$   
 $p = 0, q = 0, r = 0 \mid$   
 $p = 0, r = 1, r = 0 \mid$   
 $q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0$

## Alternative View of Tableaux: Example 2

$(\neg((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r))) = 1 \rightsquigarrow$   
 $((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r)) = 0 \rightsquigarrow$   
 $((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p \rightarrow r) = 0 \rightsquigarrow$   
 $((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$   
 $((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1, p = 0, r = 0 \rightsquigarrow$   
 $(p \rightarrow q) = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$   
 $p = 0, (p \wedge q \rightarrow r) = 1, r = 0 \mid$   
 $q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$   
 $p = 0, (p \wedge q) = 0, r = 0 \mid$   
 $p = 0, r = 1, r = 0 \mid$   
 $q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0 \rightsquigarrow$   
 $p = 0, r = 0 \mid$   
 $p = 0, q = 0, r = 0 \mid$   
 $p = 0, r = 1, r = 0 \mid$   
 $q = 1, (p \wedge q \rightarrow r) = 1, p = 0, r = 0$

The branch containing  $p = 0, r = 0$  can no more be expanded or closed so it gives us a model (in fact, two models)