Outline

Semantic Tableaux

Signed Formula

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- A signed formula is satisfiable if it has a model.

Note:

1. For every formula A and interpretation I exactly one of the signed formulas $A = 1$ and $A = 0$ is true in I.
2. A formula A is satisfiable if and only if so is the signed formula $A = 1$.

How to find a model of a signed formula?

Example: $(A \to B) = 1$.

How to find a model of a signed formula?

Operation table for $\to$:

$\to$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

Example: $(A \to B) = 1$.

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Operation table for $\rightarrow$:

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
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Example: $(A \rightarrow B) = 1$.

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$A = 0$ OR $B = 1$.

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So $(A \to B) = 1$ if and only if
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Likewise, $(A \to B) = 0$ if and only
if $A = 1$ AND $B = 0$.

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Example: $(A \rightarrow B) = 1$.

So $(A \rightarrow B) = 1$ if and only if $A = 0$ OR $B = 1$.

Likewise, $(A \rightarrow B) = 0$ if and only if $A = 1$ AND $B = 0$.

So we can use AND-OR trees to carry out case analysis.

Tableau

Tableau: a tree having signed formulas at nodes.

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Tableau for a signed formula $A = b$ has $A = b$ as a root.

Alternatively, we can regard a tableau as a collection of branches; each branch is a set of signed formulas.

Notation for a set of branches: $B_1 \mid \ldots \mid B_n$, where each of the B_i is a branch.

Branch Expansion Rules

$$
\begin{array}{rcl}
(A_1 \wedge \ldots \wedge A_n) = 0 & \rightsquigarrow & A_1 = 0 \mid \ldots \mid A_n = 0 \\
(A_1 \wedge \ldots \wedge A_n) = 1 & \rightsquigarrow & A_1 = 1, \ldots, A_n = 1 \\
\\
(A_1 \vee \ldots \vee A_n) = 0 & \rightsquigarrow & A_1 = 0, \ldots, A_n = 0 \\
(A_1 \vee \ldots \vee A_n) = 1 & \rightsquigarrow & A_1 = 1 \mid \ldots \mid A_n = 1 \\
\\
(A_1 \rightarrow A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(A_1 \rightarrow A_2) = 1 & \rightsquigarrow & A_1 = 0 \mid A_2 = 1 \\
\\
(\neg A_1) = 0 & \rightsquigarrow & A_1 = 1 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0 \\
\\
(A_1 \leftrightarrow A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 1 \mid A_1 = 1, A_2 = 0 \\
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Branch Closure Rules

These rules are introduced to mark when the set of signed formulas on a branch is unsatisfiable.

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A branch is marked closed in any of the following cases:

- it contains both $p = 0$ and $p = 1$ for some atom p

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A branch is marked closed in any of the following cases:

- it contains both $p = 0$ and $p = 1$ for some atom p
- it contains $\top = 0$;
- it contains $\bot = 1$.

A Semantic Tableau

$$(\neg(q \vee p \rightarrow p \vee q)) = 1$$

$$\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \rightarrow A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
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(A_1 \vee A_2) = 0 & \leadsto & A_1 = 0, A_2 = 0 \\
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\end{array}$$

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$$\begin{array}{c} (q \vee p) = 1 \\ (p \vee q) = 0 \end{array}$$

$$\begin{array}{c} p = 0 \\ q = 0 \end{array}$$

$$q = 1 \qquad\qquad p = 1$$

$$\begin{array}{rcl} (A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\ (A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\ (A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\ (\neg A_1) = 1 & \rightsquigarrow & A_1 = 0 \end{array}$$

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$$(q \vee p) = 1$$
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$$p = 0$$
$$q = 0$$

$q = 1$	$p = 1$
closed	

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(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
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\end{array}$$

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$$(\neg(q \vee p \to p \vee q)) = 1$$

$$(q \vee p \to p \vee q) = 0$$

$$(q \vee p) = 1$$
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$$p = 0$$
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closed   closed

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Example 2

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1$$

$$\begin{array}{rcl}
(A_1 \land A_2) = 0 & \rightsquigarrow & A_1 = 0 \mid A_2 = 0 \\
(A_1 \land A_2) = 1 & \rightsquigarrow & A_1 = 1, A_2 = 1 \\
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$$
\begin{array}{c}
(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \\
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$$(\neg p) = 1$$
$$r = 0$$

$$\begin{array}{cc} p = 0 & q = 1 \\ p = 0 & \end{array}$$

$$\begin{array}{rcl}
(A_1 \land A_2) = 0 & \rightsquigarrow & A_1 = 0 \mid A_2 = 0 \\
(A_1 \land A_2) = 1 & \rightsquigarrow & A_1 = 1, A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(A_1 \to A_2) = 1 & \rightsquigarrow & A_1 = 0 \mid A_2 = 1 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}$$

Example 2

$$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$$

$$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$$

$$((p \to q) \wedge (p \wedge q \to r)) = 1$$
$$(\neg p \to r) = 0$$

$$(p \to q) = 1$$
$$(p \wedge q \to r) = 1$$

$$(\neg p) = 1$$
$$r = 0$$

$$\begin{array}{ccc} & p = 0 & & q = 1 \\ & p = 0 & & \\ (p \wedge q) = 0 & & r = 1 & \end{array}$$

$$\begin{array}{rcl}
(A_1 \wedge A_2) = 0 & \rightsquigarrow & A_1 = 0 \mid A_2 = 0 \\
(A_1 \wedge A_2) = 1 & \rightsquigarrow & A_1 = 1, A_2 = 1 \\
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(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
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$$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$$

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$$((p \to q) \wedge (p \wedge q \to r)) = 1$$
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$$(p \to q) = 1$$
$$(p \wedge q \to r) = 1$$

$$(\neg p) = 1$$
$$r = 0$$

$$p = 0 \qquad\qquad q = 1$$

$$p = 0$$

$$(p \wedge q) = 0 \qquad\qquad r = 1$$

$$\begin{array}{rcl}
(A_1 \wedge A_2) = 0 & \rightsquigarrow & A_1 = 0 \mid A_2 = 0 \\
(A_1 \wedge A_2) = 1 & \rightsquigarrow & A_1 = 1, A_2 = 1 \\
 & & \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
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$$(p \to q) = 1$$
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$$r = 0$$

$$p = 0 \qquad\qquad q = 1$$

$$p = 0$$

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$$p = 0 \qquad\qquad q = 0$$

$$
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\end{array}
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Example 2

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1$$

$$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0$$

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$$(p \to q) = 1$$
$$(p \land q \to r) = 1$$

$$(\neg p) = 1$$
$$r = 0$$

$$p = 0 \qquad\qquad q = 1$$

$$p = 0$$

$$(p \land q) = 0 \qquad\qquad r = 1$$

$$p = 0 \qquad\qquad q = 0$$

$$\begin{array}{rcl}
(A_1 \land A_2) = 0 & \rightsquigarrow & A_1 = 0 \mid A_2 = 0 \\
(A_1 \land A_2) = 1 & \rightsquigarrow & A_1 = 1, A_2 = 1 \\
 & & \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(A_1 \to A_2) = 1 & \rightsquigarrow & A_1 = 0 \mid A_2 = 1 \\
 & & \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}$$

All rules on this branch have been applied, so the formula is satisfiable.

Finding Models Using Tableaux

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1$$

$$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0$$

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$$(p \to q) = 1$$
$$(p \land q \to r) = 1$$

$$(\neg p) = 1$$
$$r = 0$$

- $p = 0$
  - $p = 0$
    - $(p \land q) = 0$
      - $p = 0$
      - $q = 0$
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- $q = 1$

Finding Models Using Tableaux

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1$$

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$$(p \to q) = 1$$
$$(p \land q \to r) = 1$$

$$(\neg p) = 1$$
$$r = 0$$

$$p = 0 \qquad\qquad q = 1$$

$$p = 0$$

$$(p \land q) = 0 \qquad\qquad r = 1$$

$$p = 0 \qquad\qquad q = 0$$

Build an open branch on which all rules have been applied.

Finding Models Using Tableaux

$$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$$

$$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$$

$$((p \to q) \wedge (p \wedge q \to r)) = 1$$
$$(\neg p \to r) = 0$$

$$(p \to q) = 1$$
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$$(\neg p) = 1$$
$$r = 0$$

- $p = 0$
  - $p = 0$
    - $(p \wedge q) = 0$
      - $p = 0$
      - $q = 0$
    - $r = 1$
- $q = 1$

Build an open branch on which all rules have been applied.

Select signed atoms on this branch

Finding Models Using Tableaux

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1$$

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$$((p \to q) \land (p \land q \to r)) = 1$$
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$$(p \to q) = 1$$
$$(p \land q \to r) = 1$$

$$(\neg p) = 1$$
$$r = 0$$

$$p = 0 \qquad q = 1$$

$$p = 0$$

$$(p \land q) = 0 \qquad r = 1$$

$$p = 0 \qquad q = 0$$

Build an open branch on which all rules have been applied.

Select signed atoms on this branch

They give us a model

$$\{r \mapsto 0, p \mapsto 0, q \mapsto \cdots\}$$

Checking Other Properties with Tableaux

A formula A is satisfiable iff a tableau for $A = 1$ contains a complete open branch (and iff every tableau for $A = 1$ contains a complete open branch).

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Formulas A and B are equivalent iff there is a closed tableau for $(A \leftrightarrow B) = 0$ (and iff every tableau for $(A \leftrightarrow B) = 0$ is closed).

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Formulas A and B are equivalent iff there is a closed tableau for $(A \leftrightarrow B) = 0$ (and iff every tableau for $(A \leftrightarrow B) = 0$ is closed).

A fully expanded tableau for $A = 1$ gives us all models of A.

Alternative View of Tableaux

We will make the following changes:

1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
2. remove closed branches;
3. if we apply a table expansion rule to a signed formula on a branch, we will remove the formula from the branch.

$$
\begin{array}{rcl}
(A_1 \lor A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \lor A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

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3. if we apply a table expansion rule to a signed formula on a branch, we will remove the formula from the branch.

Consider Example 1 again.

$$(\neg(q \vee p \to p \vee q)) = 1$$

$$\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
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(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
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\end{array}$$

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Consider Example 1 again.

$$(\neg(q \vee p \to p \vee q)) = 1 \rightsquigarrow$$
$$(q \vee p \to p \vee q) = 0$$

$$\begin{array}{rcl} (A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\ (A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\ (A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\ (\neg A_1) = 1 & \rightsquigarrow & A_1 = 0 \end{array}$$

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1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
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$$(\neg(q \vee p \to p \vee q)) = 1 \rightsquigarrow$$
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$$\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}$$

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1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
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Consider Example 1 again.

$$(\neg(q \vee p \to p \vee q)) = 1 \rightsquigarrow$$
$$(q \vee p \to p \vee q) = 0 \rightsquigarrow$$
$$(q \vee p) = 1, (p \vee q) = 0$$

$$\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
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1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
2. remove closed branches;
3. if we apply a table expansion rule to a signed formula on a branch, we will remove the formula from the branch.

Consider Example 1 again.

$$(\neg(q \lor p \to p \lor q)) = 1 \leadsto$$
$$(q \lor p \to p \lor q) = 0 \leadsto$$
$$(q \lor p) = 1, (p \lor q) = 0$$

$$\begin{array}{rcl}
(A_1 \lor A_2) = 0 & \leadsto & A_1 = 0, A_2 = 0 \\
(A_1 \lor A_2) = 1 & \leadsto & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \leadsto & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \leadsto & A_1 = 0
\end{array}$$

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We will make the following changes:

1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
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3. if we apply a table expansion rule to a signed formula on a branch, we will remove the formula from the branch.

Consider Example 1 again.

$$
\begin{array}{l}
(\neg(q \lor p \to p \lor q)) = 1 \rightsquigarrow \\
(q \lor p \to p \lor q) = 0 \rightsquigarrow \\
(q \lor p) = 1, (p \lor q) = 0 \rightsquigarrow \\
(q \lor p) = 1, p = 0, q = 0
\end{array}
$$

$$
\begin{array}{rcl}
(A_1 \lor A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \lor A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
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$$
\begin{array}{rcl}
(A_1 \lor A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \lor A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

$(\neg(q \lor p \to p \lor q)) = 1 \rightsquigarrow$
$(q \lor p \to p \lor q) = 0 \rightsquigarrow$
$(q \lor p) = 1, (p \lor q) = 0 \rightsquigarrow$
$(q \lor p) = 1, p = 0, q = 0$

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We will make the following changes:

1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
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\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

$(\neg(q \vee p \to p \vee q)) = 1 \rightsquigarrow$
$(q \vee p \to p \vee q) = 0 \rightsquigarrow$
$(q \vee p) = 1, (p \vee q) = 0 \rightsquigarrow$
$(q \vee p) = 1, p = 0, q = 0 \rightsquigarrow$
$q = 1, p = 0, q = 0 \mid p = 1, p = 0, q = 0$

Alternative View of Tableaux

We will make the following changes:

1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
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3. if we apply a table expansion rule to a signed formula on a branch, we will remove the formula from the branch.

Consider Example 1 again.

$$
\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

$(\neg(q \vee p \to p \vee q)) = 1 \rightsquigarrow$
$(q \vee p \to p \vee q) = 0 \rightsquigarrow$
$(q \vee p) = 1, (p \vee q) = 0 \rightsquigarrow$
$(q \vee p) = 1, p = 0, q = 0 \rightsquigarrow$
$q = 1, p = 0, q = 0 \mid p = 1, p = 0, q = 0$

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We will make the following changes:

1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
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Consider Example 1 again.

$$
\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

$$
\begin{array}{l}
(\neg(q \vee p \to p \vee q)) = 1 \rightsquigarrow \\
(q \vee p \to p \vee q) = 0 \rightsquigarrow \\
(q \vee p) = 1, (p \vee q) = 0 \rightsquigarrow \\
(q \vee p) = 1, p = 0, q = 0 \rightsquigarrow \\
q = 1, p = 0, q = 0 \mid p = 1, p = 0, q = 0 \rightsquigarrow \\
p = 1, p = 0, q = 0
\end{array}
$$

Alternative View of Tableaux

We will make the following changes:

1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
2. remove closed branches;
3. if we apply a table expansion rule to a signed formula on a branch, we will remove the formula from the branch.

Consider Example 1 again.

$$
\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \rightarrow A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

$$
\begin{array}{l}
(\neg(q \vee p \rightarrow p \vee q)) = 1 \rightsquigarrow \\
(q \vee p \rightarrow p \vee q) = 0 \rightsquigarrow \\
(q \vee p) = 1, (p \vee q) = 0 \rightsquigarrow \\
(q \vee p) = 1, p = 0, q = 0 \rightsquigarrow \\
q = 1, p = 0, q = 0 \mid p = 1, p = 0, q = 0 \rightsquigarrow \\
p = 1, p = 0, q = 0
\end{array}
$$

Alternative View of Tableaux

We will make the following changes:

1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
2. remove closed branches;
3. if we apply a table expansion rule to a signed formula on a branch, we will remove the formula from the branch.

Consider Example 1 again.

$$
\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

$$
\begin{array}{l}
(\neg(q \vee p \to p \vee q)) = 1 \rightsquigarrow \\
(q \vee p \to p \vee q) = 0 \rightsquigarrow \\
(q \vee p) = 1, (p \vee q) = 0 \rightsquigarrow \\
(q \vee p) = 1, p = 0, q = 0 \rightsquigarrow \\
q = 1, p = 0, q = 0 \mid p = 1, p = 0, q = 0 \rightsquigarrow \\
p = 1, p = 0, q = 0
\end{array}
$$

All branches are closed, so the signed formula $(\neg(q \vee p \to p \vee q)) = 1$ is unsatisfiable.

Alternative View of Tableaux: Example 2

$$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$$

Alternative View of Tableaux: Example 2

$$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$$

Alternative View of Tableaux: Example 2

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0$$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$

$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$

$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0$

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$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
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$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p) = 1, r = 0$

Alternative View of Tableaux: Example 2

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r)) = 1, (\neg p) = 1, r = 0$$

Alternative View of Tableaux: Example 2

$$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$$
$$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$$
$$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$$
$$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$$
$$((p \to q) \wedge (p \wedge q \to r)) = 1, p = 0, r = 0$$

Alternative View of Tableaux: Example 2

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r)) = 1, p = 0, r = 0$$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, p = 0, r = 0 \rightsquigarrow$
$(p \to q) = 1, (p \wedge q \to r) = 1, p = 0, r = 0$

Alternative View of Tableaux: Example 2

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$$
$$((p \to q) \land (p \land q \to r)) = 1, p = 0, r = 0 \rightsquigarrow$$
$$(p \to q) = 1, (p \land q \to r) = 1, p = 0, r = 0$$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, p = 0, r = 0 \rightsquigarrow$
$(p \to q) = 1, (p \wedge q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \wedge q \to r) = 1, r = 0 \mid$
$q = 1, (p \wedge q \to r) = 1, p = 0, r = 0$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$
$((p \to q) \land (p \land q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$
$((p \to q) \land (p \land q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$
$((p \to q) \land (p \land q \to r)) = 1, p = 0, r = 0 \rightsquigarrow$
$(p \to q) = 1, (p \land q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \land q \to r) = 1, r = 0 \mid$
$q = 1, (p \land q \to r) = 1, p = 0, r = 0$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, p = 0, r = 0 \rightsquigarrow$
$(p \to q) = 1, (p \wedge q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \wedge q \to r) = 1, r = 0 \mid$
$q = 1, (p \wedge q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \wedge q) = 0, r = 0 \mid$
$p = 0, r = 1, r = 0 \mid$
$q = 1, (p \wedge q \to r) = 1, p = 0, r = 0$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$
$((p \to q) \land (p \land q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$
$((p \to q) \land (p \land q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$
$((p \to q) \land (p \land q \to r)) = 1, p = 0, r = 0 \rightsquigarrow$
$(p \to q) = 1, (p \land q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \land q \to r) = 1, r = 0 \mid$
$q = 1, (p \land q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \land q) = 0, r = 0 \mid$
$p = 0, r = 1, r = 0 \mid$
$q = 1, (p \land q \to r) = 1, p = 0, r = 0$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, p = 0, r = 0 \rightsquigarrow$
$(p \to q) = 1, (p \wedge q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \wedge q \to r) = 1, r = 0 \mid$
$q = 1, (p \wedge q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \wedge q) = 0, r = 0 \mid$
$p = 0, r = 1, r = 0 \mid$
$q = 1, (p \wedge q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, r = 0 \mid$
$p = 0, q = 0, r = 0 \mid$
$p = 0, r = 1, r = 0 \mid$
$q = 1, (p \wedge q \to r) = 1, p = 0, r = 0$

Alternative View of Tableaux: Example 2

$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p) = 1, r = 0 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r)) = 1, p = 0, r = 0 \rightsquigarrow$
$(p \to q) = 1, (p \wedge q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \wedge q \to r) = 1, r = 0 \mid$
$q = 1, (p \wedge q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, (p \wedge q) = 0, r = 0 \mid$
$p = 0, r = 1, r = 0 \mid$
$q = 1, (p \wedge q \to r) = 1, p = 0, r = 0 \rightsquigarrow$
$p = 0, r = 0 \mid$
$p = 0, q = 0, r = 0 \mid$
$p = 0, r = 1, r = 0 \mid$
$q = 1, (p \wedge q \to r) = 1, p = 0, r = 0$

The branch containing $p = 0, r = 0$ can no more be expanded or closed so it gives us a model (in fact, two models)