

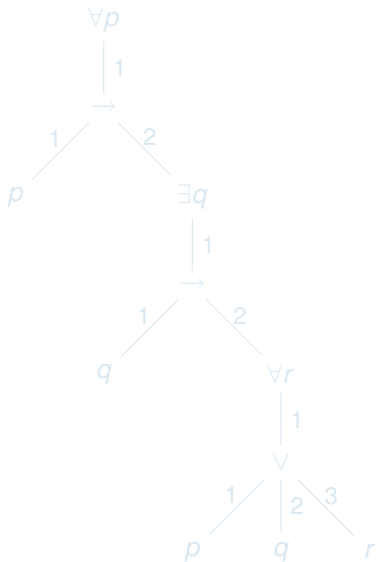
Exercise 7. Problem 1

Draw the parse tree for the following formula:

$$\forall p(p \rightarrow \exists q(q \rightarrow \forall r(p \vee q \vee r))).$$

Underline all free occurrences of variables in this formula.

There are no free variables in this formula.



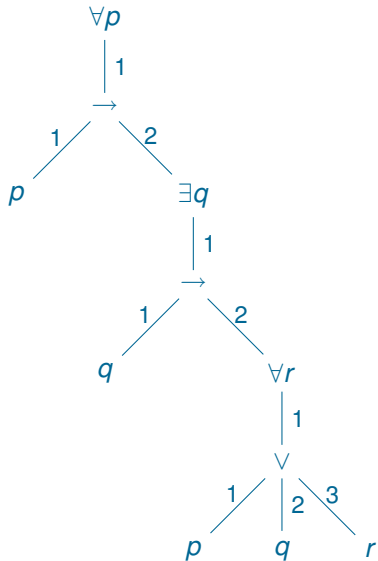
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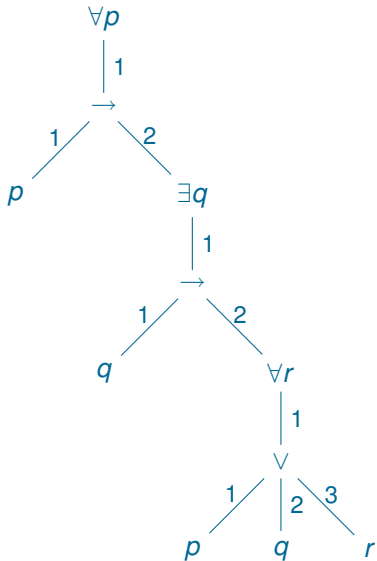
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There are no free variables in this formula.



Common Mistakes

- ▶ A problem with parsing disjunctions: two branches are drawn instead of three.
- ▶ A problem with identifying bound variables, some do not notice that the quantifiers over p and q make the second appearances of p and q bound and conclude they are free.

Exercise 7. Problem 2

Transform the following formula into prenex form:

$$\forall p \neg p \vee \forall p p \rightarrow \neg p.$$

Solution

This formula is not rectified so we should rectify it first:

$$\begin{aligned} & \forall p \neg p \vee \forall p p \rightarrow \neg p \Rightarrow \\ & \forall p_1 \neg p_1 \vee \forall p p \rightarrow \neg p \Rightarrow \\ & \forall p_1 \neg p_1 \vee \forall p_2 p_2 \rightarrow \neg p. \end{aligned}$$

Then we can apply prenexing transformation as follows:

$$\begin{aligned} & \forall p_1 \neg p_1 \vee \forall p_2 p_2 \rightarrow \neg p \Rightarrow \\ & \forall p_1 (\neg p_1 \vee \forall p_2 p_2) \rightarrow \neg p \Rightarrow \\ & \exists p_1 (\neg p_1 \vee \forall p_2 p_2 \rightarrow \neg p) \Rightarrow \\ & \exists p_1 (\forall p_2 (\neg p_1 \vee p_2) \rightarrow \neg p) \Rightarrow \\ & \exists p_1 \exists p_2 (\neg p_1 \vee p_2 \rightarrow \neg p) \end{aligned}$$

Common Mistakes

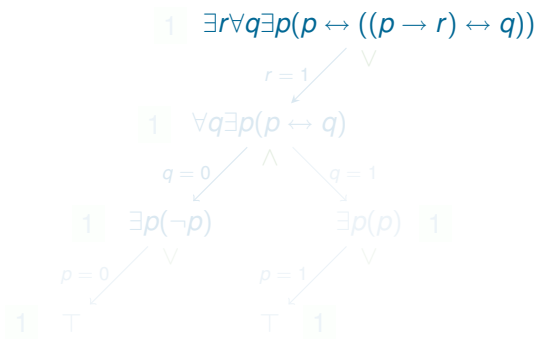
- ▶ Lots of problems with putting parentheses properly, therefore the formula is prenexed wrongly.
- ▶ No rectification is done in very many cases. Sometimes it is not done at all. Some rename free variables instead of bound: this is a mistake since it changes semantics of the formula.
- ▶ The last step is done incorrectly, i.e. the second quantifier is not changed to existential, while the first one is.
- ▶ Some other problem with prenexing rules.

Exercise 7. Problem 3

Evaluate the following formula using the Splitting Algorithm:

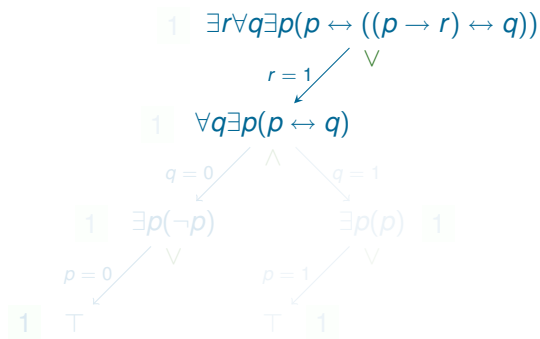
$$\exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)).$$

Solution



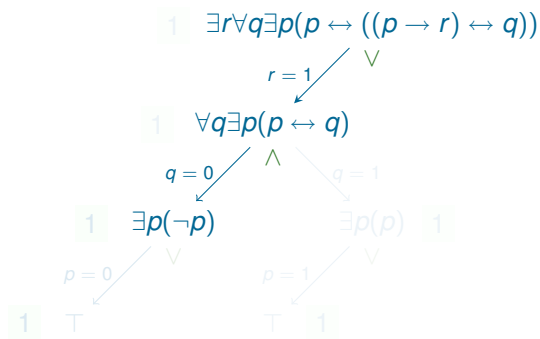
Therefore, the formula is true

Solution



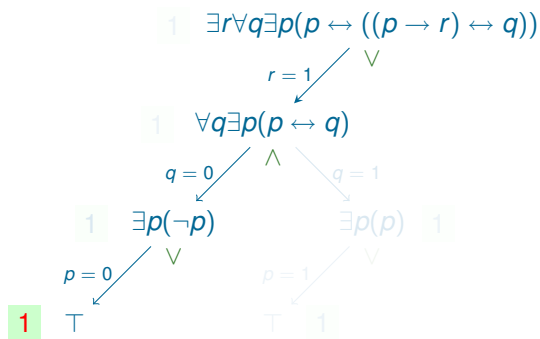
Therefore, the formula is true

Solution



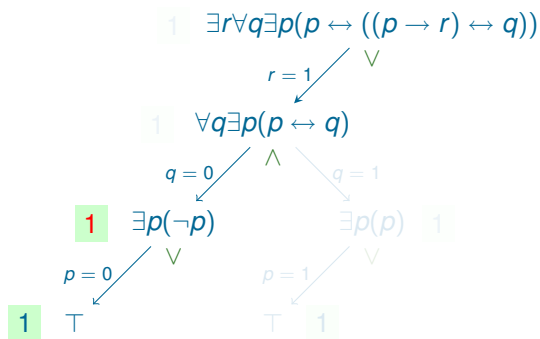
Therefore, the formula is true

Solution



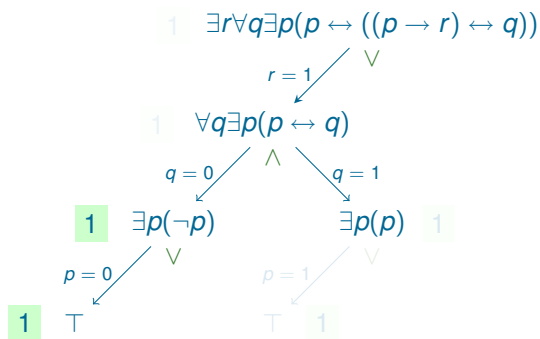
Therefore, the formula is true

Solution



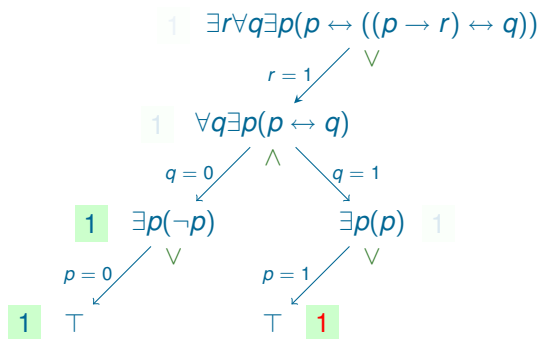
Therefore, the formula is true

Solution



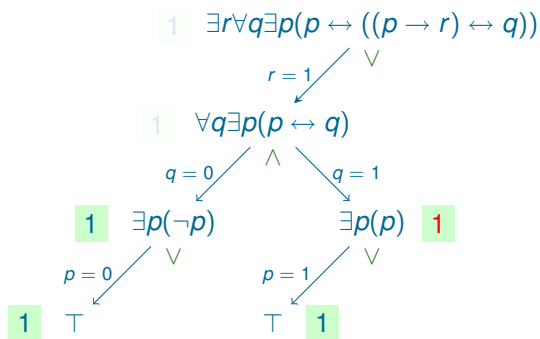
Therefore, the formula is true

Solution



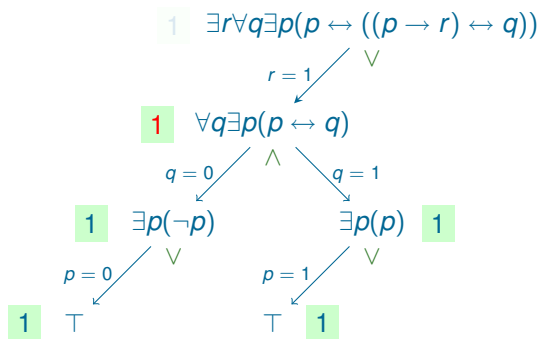
Therefore, the formula is true

Solution



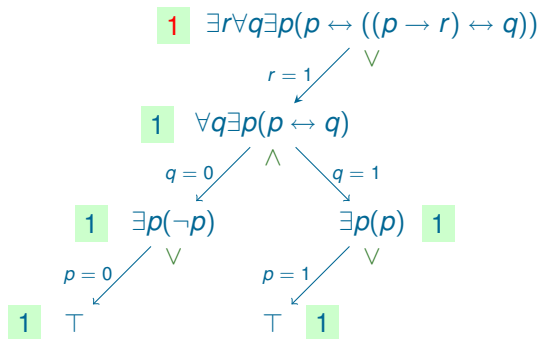
Therefore, the formula is true

Solution



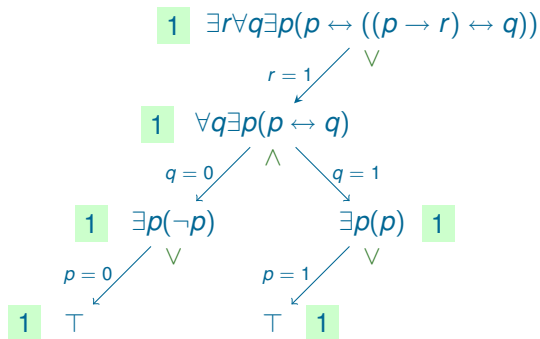
Therefore, the formula is true

Solution



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Therefore, the formula is true

Common Mistakes

- ▶ A splitting variable is not chosen from the outermost prefix: the first split is done on the variable q .
- ▶ Some draw the splitting tree but do not write whether the formula is true or not.
- ▶ Some do not understand the definitions of validity and satisfiability and try to build models for a closed formula using the splitting tree.