

Exercise 8. Problem 1

Evaluate the following formula using only the pure literal rule, universal literal deletion and unit propagation.

$$\begin{aligned} & \forall p \exists q \forall s \exists r ((p \vee q \vee s) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee r \vee \neg s)) \Rightarrow \\ & \exists q \forall s \exists r ((q \vee s) \wedge (\neg q \vee \neg r) \wedge (\neg q \vee r \vee \neg s)) \Rightarrow \\ & \exists q \forall s \exists r (q \wedge (\neg q \vee \neg r) \wedge (\neg q \vee r \vee \neg s)) \Rightarrow \\ & \forall s \exists r (\neg r \wedge (r \vee \neg s)) \Rightarrow \\ & \forall s (\neg s) \Rightarrow \\ & \perp \end{aligned}$$

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The literal p is pure. Since p is bound by a universal quantifier, we delete p from the each clause in which it occurs.

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We can apply universal literal deletion to s in the first clause.

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Now we can apply unit propagation to q .

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Now we can apply unit propagation to $\neg r$.

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Now we can apply unit propagation to $\neg s$.

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Common Mistakes

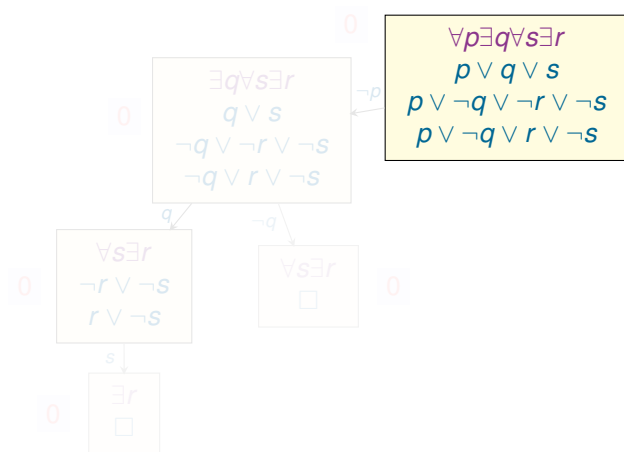
- ▶ A mistake when applying universal literal deletion: $\neg s$ was deleted from the last clause.

Exercise 8. Problem 2

Evaluate the following formula using DPLL:

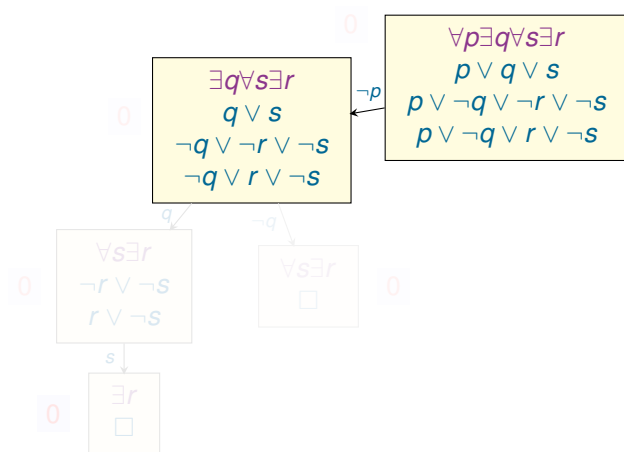
$$\forall p \exists q \forall s \exists r ((p \vee q \vee s) \wedge (p \vee \neg q \vee \neg r \vee \neg s) \wedge (p \vee \neg q \vee r \vee \neg s))$$

Solution



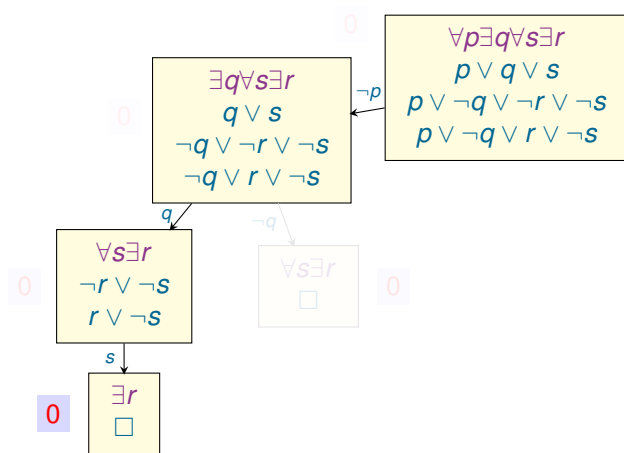
Thus, the formula is false.

Solution



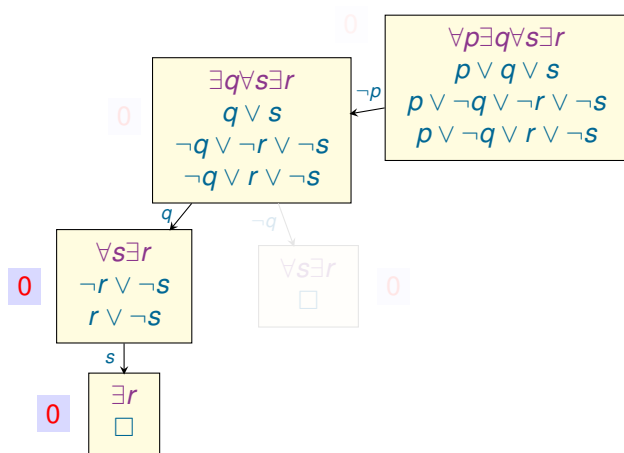
Thus, the formula is false.

Solution



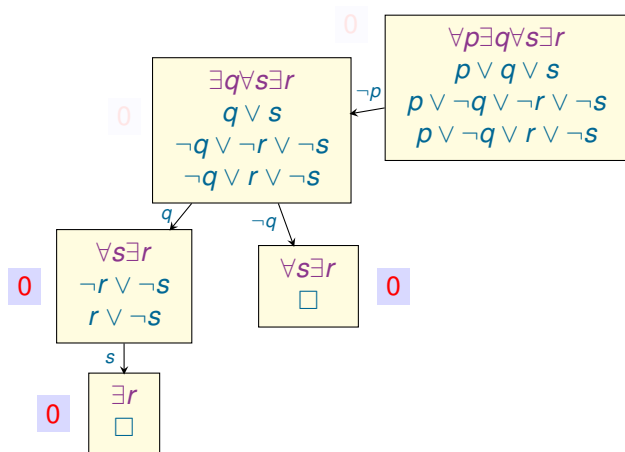
Thus, the formula is false.

Solution



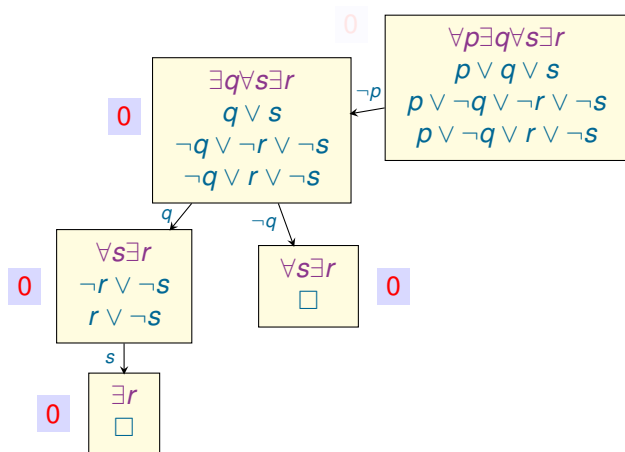
Thus, the formula is false.

Solution



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Solution



Thus, the formula is false.

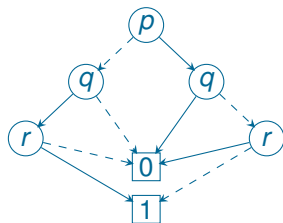
Common Mistakes

- ▶ The main problem was with branching on the variable q . Only one branch was considered.
- ▶ Some applied pure literal rule: the question was to use DPLL.

Exercise 8. Problem 3

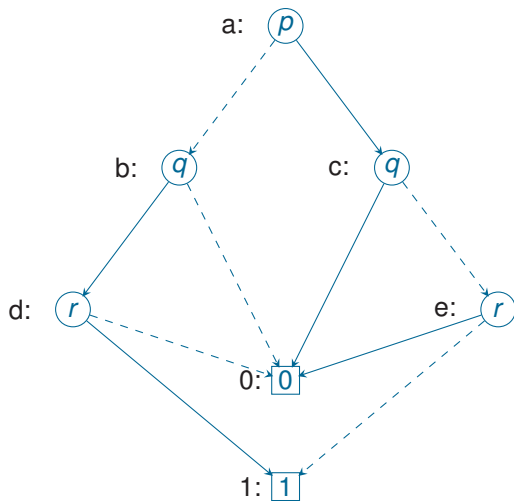
A formula F has the OBDD shown on the right.

Apply the quantification algorithm to this OBDD to obtain the OBDD for the formula $\exists q F$.

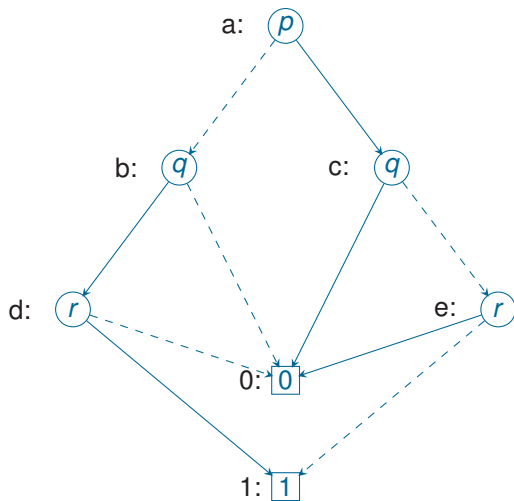
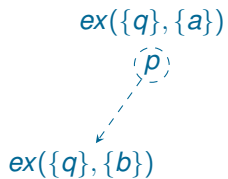


Solution

$ex(\{q\}, \{a\})$

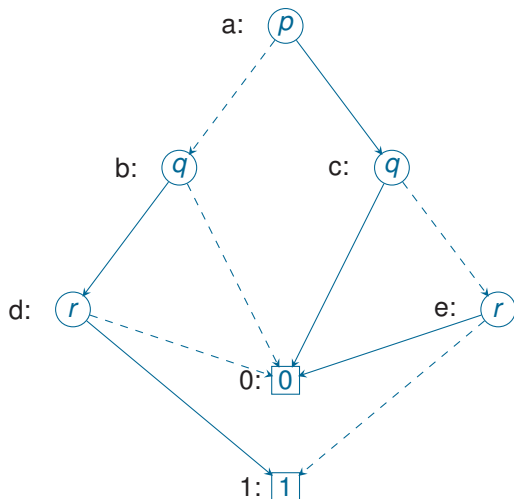


Solution



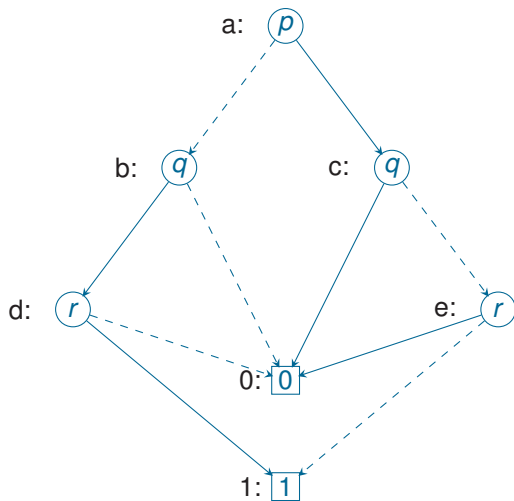
Solution

$ex(\{q\}, \{a\})$
 $ex(\{q\}, \{b\})$
 $ex(\{\}, \{0, d\})$



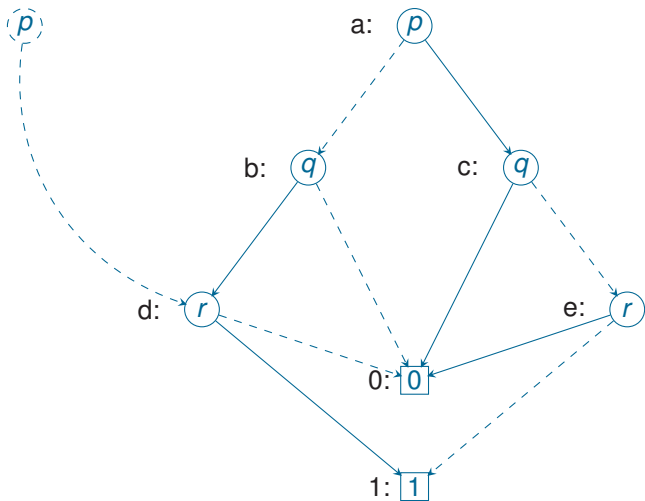
Solution

$ex(\{q\}, \{a\})$
 $ex(\{q\}, \{b\})$
 $ex(\{\}, \{0, d\})$
 $ex(\{\}, \{d\})$

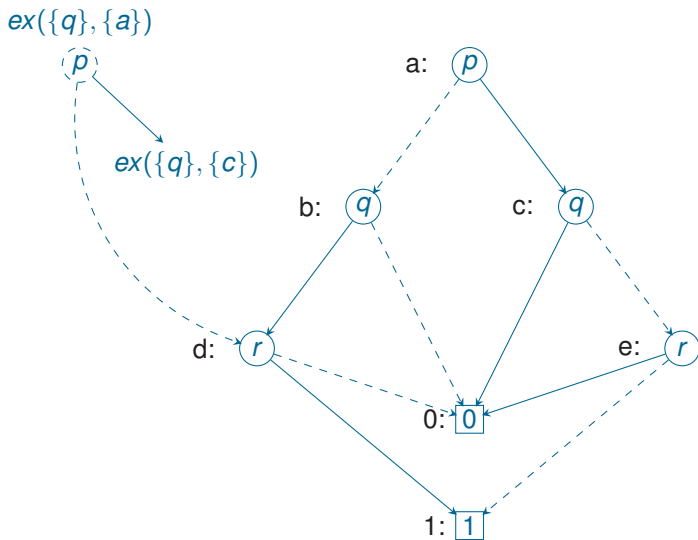


Solution

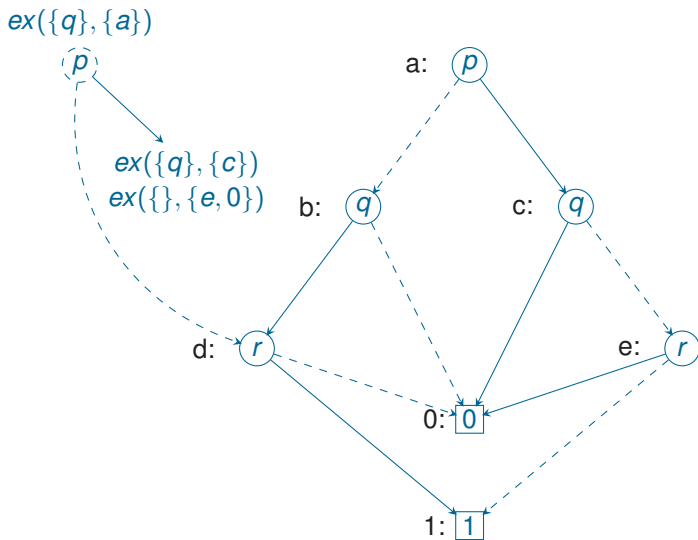
$ex(\{q\}, \{a\})$



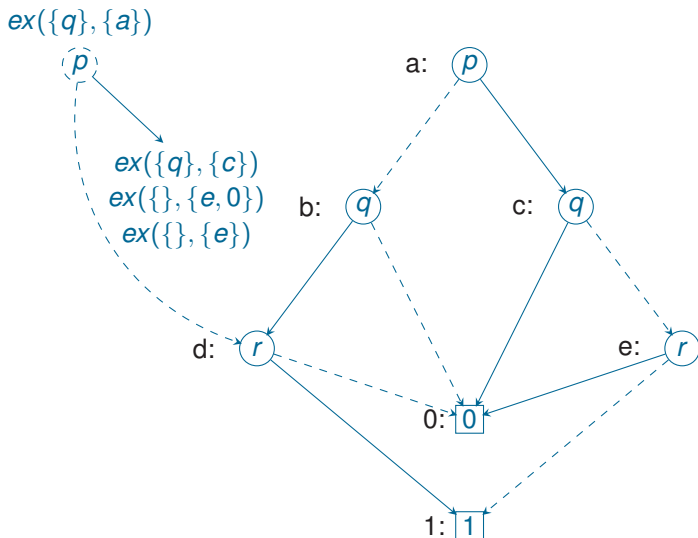
Solution



Solution

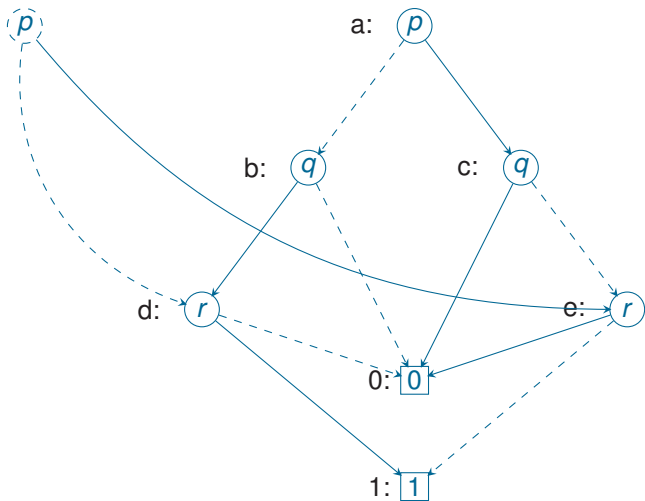


Solution



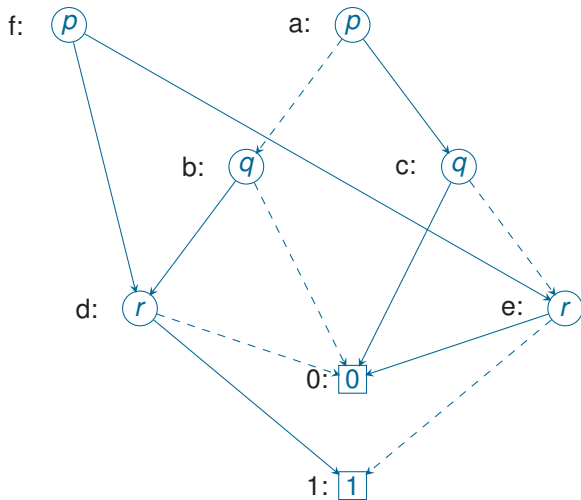
Solution

$ex(\{q\}, \{a\})$



Solution

$$ex(\{q\}, \{a\}) = f$$



Common Mistakes

- ▶ Some built the formula represented by this OBDD first and then building OBDD for the quantified formula by considering models for both cases $q = 0$ and $q = 1$. The answer was right but the procedure not applied.
- ▶ Some omit the very first procedure call $ex(\{q\}, \{a\})$.
- ▶ Some completed the list of the procedure calls but had difficulties to build the OBDD afterwards.
- ▶ Many do not integrate the resulting OBDD in the global dag.