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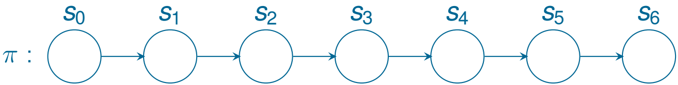

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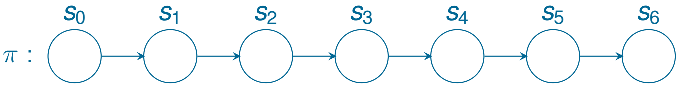

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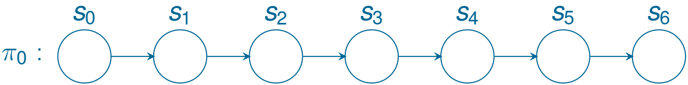

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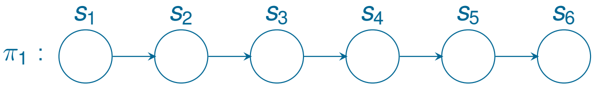

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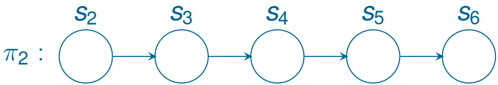

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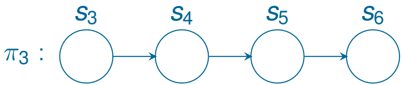

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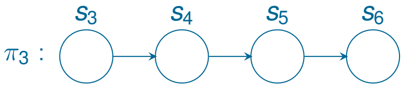

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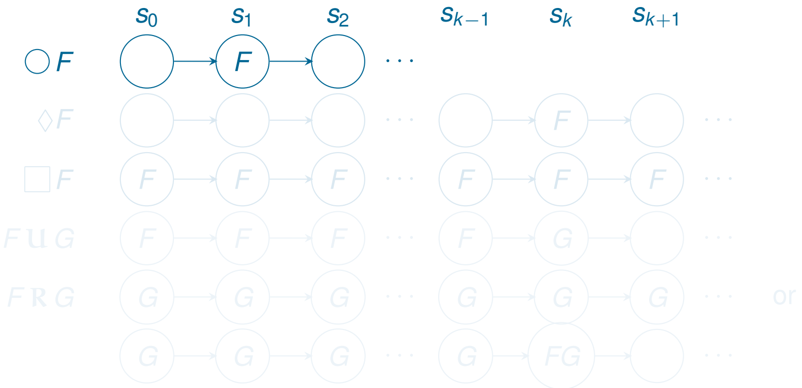

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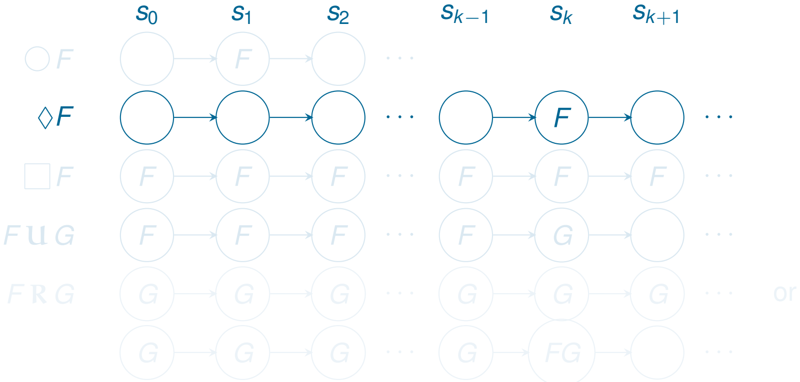

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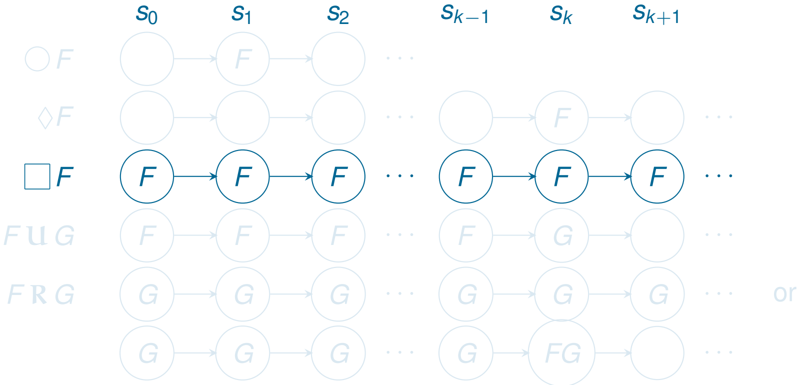

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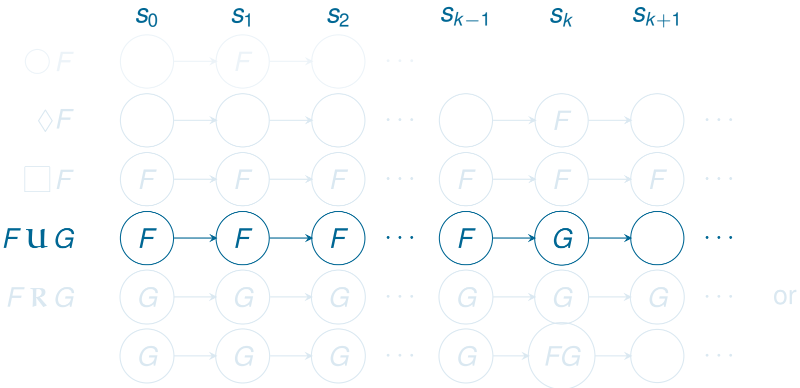

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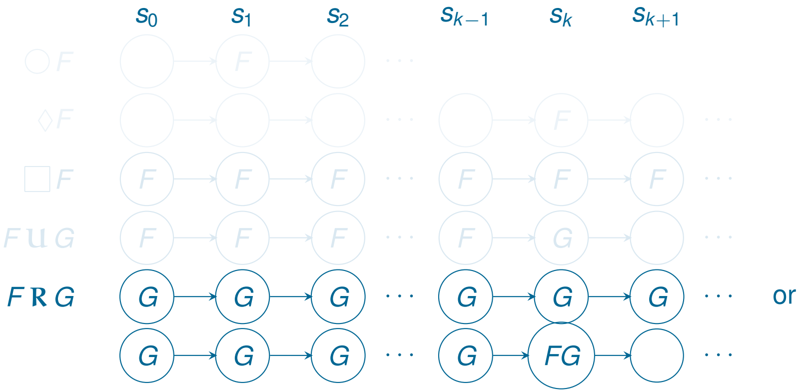

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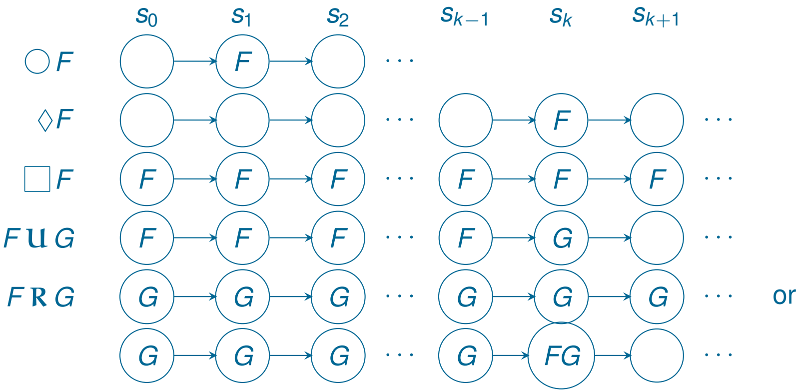

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Two LTL formulas F and G are called equivalent, denoted $F \equiv G$, if for every path π we have $\pi \models F$ if and only if $\pi \models G$.

We are not interested in satisfiability, validity etc. for temporal formulas.

For an LTL formula F we can consider two kinds of properties of $\mathbb{S}$:

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Precedences of Connectives and Temporal Operators

Connective	Precedence
$\neg, \bigcirc, \Diamond, \Box$	5
$\mathcal{U}, \mathcal{R}$	4
$\wedge, \vee$	3
$\rightarrow$	2
$\leftrightarrow$	1

Expressing Some Properties

1. F never holds in two consecutive states.
2. If F holds in a state s, it also holds at all states after s.
3. F holds in most one state.
4. F holds in least two states.
5. If F holds in state s_i, then G must hold in at least one of the two states just before s_i, that is s_{i-1} and s_{i-2}.
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Meaning of Some Formulas

- $\Box\Diamond F$;
- $\Diamond\Box F$;
- $\Box(F \rightarrow \bigcirc F)$;
- $\neg F \,\mathcal{U}\, \Box F$;
- $F \,\mathcal{U}\, \neg F$;
- $\Diamond F \wedge \Box(F \rightarrow \bigcirc F)$;

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- $\Diamond F \land \Box(F \to \bigcirc F)$;

Equivalences: Unwinding Properties

$$\begin{aligned}
\Diamond F &\equiv F \vee \bigcirc \Diamond F \\
\Box F &\equiv F \wedge \bigcirc \Box F \\
F \,\mathcal{U}\, G &\equiv G \vee (F \wedge \bigcirc (F \,\mathcal{U}\, G)) \\
F \,\mathcal{R}\, G &\equiv G \wedge (F \vee \bigcirc (F \,\mathcal{R}\, G))
\end{aligned}$$

Equivalences: Negation of Temporal Operators

$$
\begin{aligned}
\neg \bigcirc F &\equiv \bigcirc \neg F \\
\neg \Diamond F &\equiv \Box \neg F \\
\neg \Box F &\equiv \Diamond \neg F \\
\neg (F \, \mathcal{U} \, G) &\equiv \neg F \, \mathcal{R} \, \neg G \\
\neg (F \, \mathcal{R} \, G) &\equiv \neg F \, \mathcal{U} \, \neg G
\end{aligned}
$$

Expressing Temporal Operators Using $\mathcal{U}$

$$
\begin{array}{rcl}
\Diamond F & \equiv & \top \, \mathcal{U} \, F \\
\Box F & \equiv & \neg(\top \, \mathcal{U} \, \neg F) \\
F \, \mathcal{R} \, G & \equiv & \neg(\neg F \, \mathcal{U} \, \neg G).
\end{array}
$$

Therefore, all operators can be expressed using $\bigcirc$ and $\mathcal{U}$.

Other Equivalences

$$\begin{aligned} \Diamond(F \vee G) &\equiv \Diamond F \vee \Diamond G \\ \Box(F \wedge G) &\equiv \Box F \wedge \Box G \end{aligned}$$

But

$$\begin{aligned} \Box(F \vee G) &\not\equiv \Box F \vee \Box G \\ \Diamond(F \wedge G) &\not\equiv \Diamond F \wedge \Diamond G \end{aligned}$$

How to Show that Two Formulas are not Equivalent?

Find a path that satisfies one of the formulas but not the other. For example for $\Box(F \lor G)$ and $\Box F \lor \Box G$.