Outline

Random Clause Generation

How can generate a random clause?

Let's first generate a random literal.

A random clause is a collection of random literals.

- Fix a number n of boolean variables;
- Select a literal among $p_1, \ldots, p_n, \neg p_1, \ldots, \neg p_n$ with an equal probability.
- Fix the length k of the clause;

Suppose we generate random clauses one after one. How does the set of models of this set change?

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Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 32

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \lor \neg p_3$

$\neg p_2 \lor p_1$

$\neg p_2 \lor p_2$

$p_1 \lor p_1$

$\neg p_5 \lor p_5$

$p_4 \lor p_5$

$\neg p_5 \lor \neg p_3$

$p_2 \lor \neg p_4$

$p_5 \lor \neg p_2$

$p_5 \lor p_2$

$\neg p_1 \lor \neg p_4$

$p_5 \lor p_2$

$\neg p_1 \lor \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

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$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 24

This set of 13 clauses is unsatisfiable.

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$\neg p_2 \vee \neg p_3$

$\neg p_2 \vee p_1$

$\neg p_2 \vee p_2$

$p_1 \vee p_1$

$\neg p_5 \vee p_5$

$p_4 \vee p_5$

$\neg p_5 \vee \neg p_3$

$p_2 \vee \neg p_4$

$p_5 \vee \neg p_2$

$p_5 \vee p_2$

$\neg p_1 \vee \neg p_4$

$p_5 \vee p_2$

$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

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$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

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$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
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p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
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$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
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$p_4 \vee p_5$
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$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
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$p_5 \vee p_2$
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p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 12

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

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$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
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$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 9

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 9

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Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 7

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 7

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 4

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 4

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \lor \neg p_3$
$\neg p_2 \lor p_1$
$\neg p_2 \lor p_2$
$p_1 \lor p_1$
$\neg p_5 \lor p_5$
$p_4 \lor p_5$
$\neg p_5 \lor \neg p_3$
$p_2 \lor \neg p_4$
$p_5 \lor \neg p_2$
$p_5 \lor p_2$
$\neg p_1 \lor \neg p_4$
$p_5 \lor p_2$
$\neg p_1 \lor \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 3

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 3

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 1

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 1

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Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 1

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Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 1

This set of 13 clauses is unsatisfiable.

Example (obtained by a program) for $n = 5$ and $k = 2$

$\neg p_2 \vee \neg p_3$
$\neg p_2 \vee p_1$
$\neg p_2 \vee p_2$
$p_1 \vee p_1$
$\neg p_5 \vee p_5$
$p_4 \vee p_5$
$\neg p_5 \vee \neg p_3$
$p_2 \vee \neg p_4$
$p_5 \vee \neg p_2$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_4$
$p_5 \vee p_2$
$\neg p_1 \vee \neg p_5$

p_1	p_2	p_3	p_4	p_5
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
0	1	0	0	0
0	1	0	0	1
0	1	0	1	0
0	1	0	1	1
0	1	1	0	0
0	1	1	0	1
0	1	1	1	0
0	1	1	1	1

p_1	p_2	p_3	p_4	p_5
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Number of models: 0

This set of 13 clauses is unsatisfiable.

Random Clause Generation

We are interested in the probability that a set of clauses of a given size is unsatisfiable.

Fix:

- Number n of boolean variables;
- Number k of literals per clause, so we will generate k-SAT instances;
-

Generate clauses, each one has k literals randomly generated among $p_1, \dots, p_n, \neg p_1, \dots, \neg p_n$ with an equal probability.

Note that the probability is a monotone function: the more clauses we generate, the higher chance we have that the set is unsatisfiable.

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Random Clause Generation

We are interested in the probability that a set of clauses of a given size is unsatisfiable.

Fix:

- Number n of boolean variables;
- Number k of literals per clause, so we will generate k-SAT instances;
- Number m of the clauses.

Generate clauses, each one has k literals randomly generated among $p_1, \ldots, p_n, \neg p_1, \ldots, \neg p_n$ with an equal probability.

Note that the probability is a monotone function: the more clauses we generate, the higher chance we have that the set is unsatisfiable.

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We are interested in the probability that a set of clauses of a given size is unsatisfiable.

Fix:

- Number n of boolean variables;
- Number k of literals per clause, so we will generate k-SAT instances;
- Number m of the clauses.

Generate m clauses, each one has k literals randomly generated among $p_1, \ldots, p_n, \neg p_1, \ldots, \neg p_n$ with an equal probability.

Note that the probability is a monotone function: the more clauses we generate, the higher chance we have that the set is unsatisfiable.

Random Clause Generation

We are interested in the probability that a set of clauses of a given size is unsatisfiable.

Fix:

- Number n of boolean variables;
- Number k of literals per clause, so we will generate k-SAT instances;
- Number m of the clauses.

Generate m clauses, each one has k literals randomly generated among $p_1, \ldots, p_n, \neg p_1, \ldots, \neg p_n$ with an equal probability.

Note that the probability is a monotone function: the more clauses we generate, the higher chance we have that the set is unsatisfiable.

Random Clause Generation

We are interested in the probability that a set of clauses of a given size is unsatisfiable.

Fix:

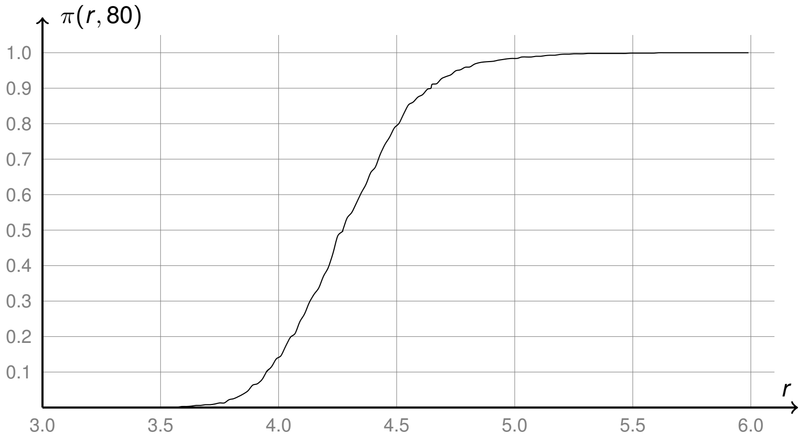
- Number n of boolean variables;
- Number k of literals per clause, so we will generate k-SAT instances;
- Real number r: ratio of clauses per variable.

Generate $[rn]$ clauses, each one has k literals randomly generated among $p_1, \ldots, p_n, \neg p_1, \ldots, \neg p_n$ with an equal probability.

Note that the probability is a monotone function: the more clauses we generate, the higher chance we have that the set is unsatisfiable.

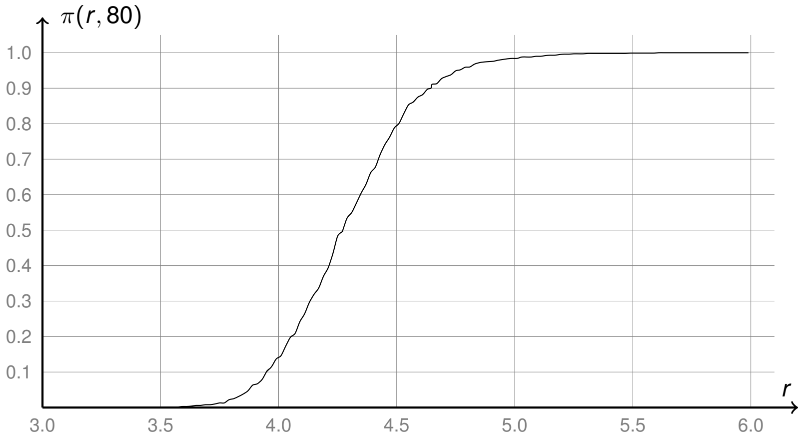
Probability of obtaining an unsatisfiable set

Crossover point: the value of r at which the probability crosses 0.5.



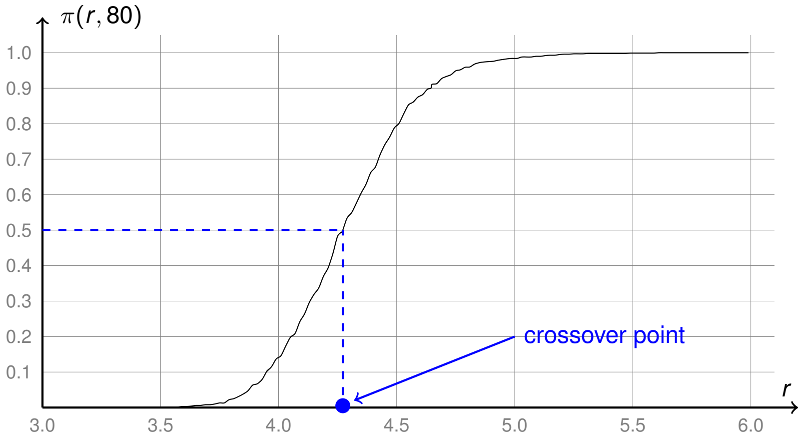
Probability of obtaining an unsatisfiable set

Crossover point: the value of r at which the probability crosses 0.5.



Probability of obtaining an unsatisfiable set

Crossover point: the value of r at which the probability crosses 0.5.



ϵ-window

Take a (small) number $\epsilon > 0$. ϵ-window is the interval of values of r where the probability is between ϵ and $1 - \epsilon$.

For example, take $\epsilon = 0.1$.

ϵ-window

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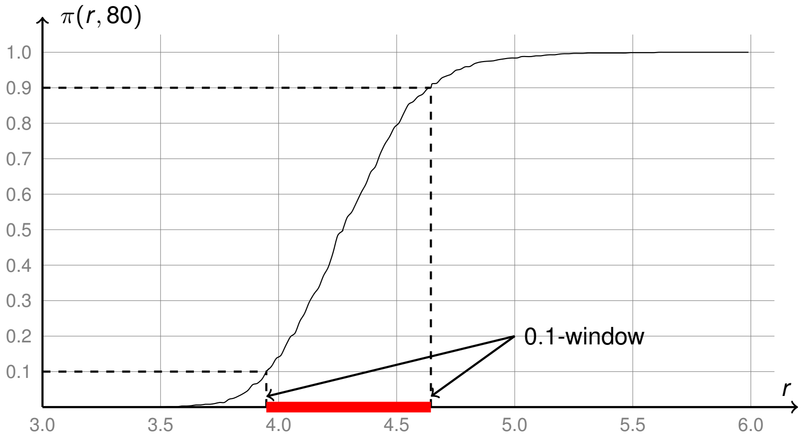
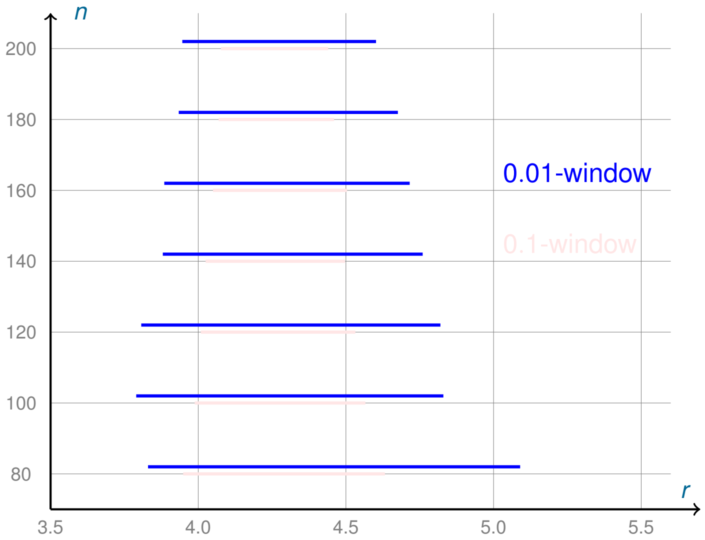


Scaling Window Effect



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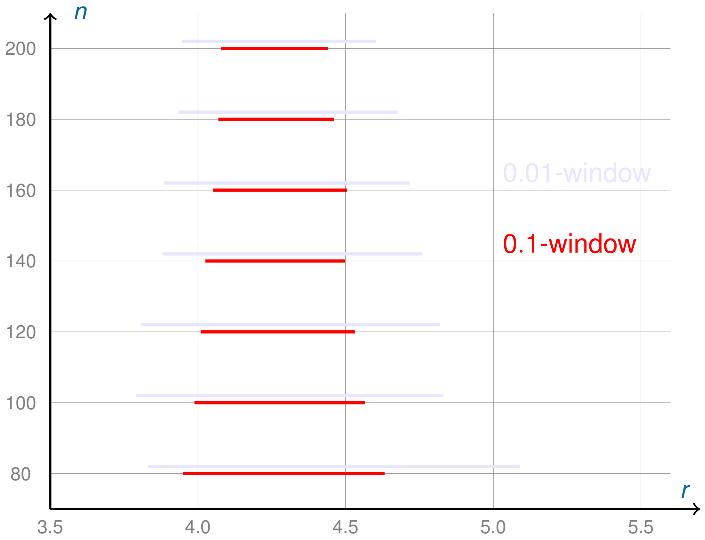

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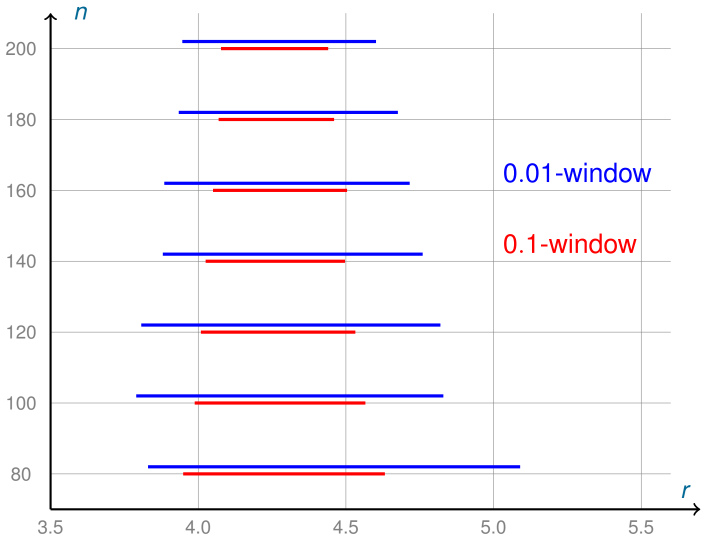
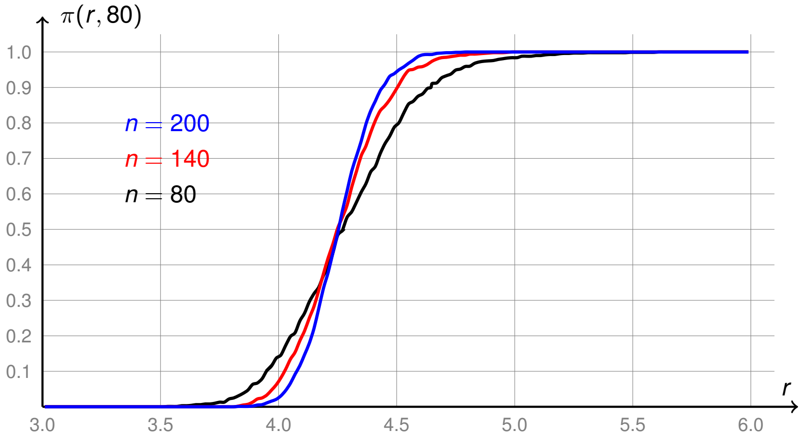
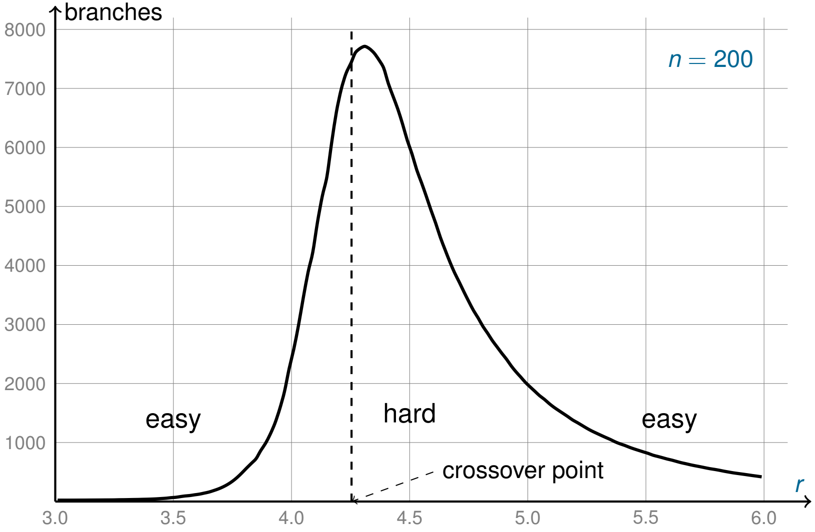


Sharp Phase Transition



Easy-Hard-Easy Pattern



Satisfiability Algorithm that Cannot Establish Unsatisfiability

procedure *CHAOS*(*S*)
input: set of clauses *S*
output: interpretation *I* such that $I \models S$ or *don't know*
parameters: positive integer *MAX-TRIES*
begin
  repeat *MAX-TRIES* times
    I := random interpretation
    if $I \models S$ **then return** I
  return *don't know*
end

Satisfiability has short witnesses: interpretations.

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