Randomised Algorithms for SAT

- Choose a random interpretation.
- If this interpretation is not a model, repeatedly choose a variable and change its value in the interpretation (flip the variable).

The flipped variables are chosen using heuristics or randomly, or both.

$$flip(I,p)(q) = \begin{cases} I(q), & \text{if } p \neq q; \\ 1, & \text{if } p = q \text{ and } I(p) = 0; \\ 0, & \text{if } p = q \text{ and } I(p) = 1. \end{cases}$$

In other words, the interpretation $flip(I,p)$ is obtained from I by changing its value on p.

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In other words, the interpretation $flip(I,p)$ is obtained from I by changing its value on p.

GSAT

procedure *GSAT*(S)
input: set of clauses S
output: interpretation I such that $I \models S$ or *don't know*
parameters: integers *MAX-TRIES*, *MAX-FLIPS*
begin
  repeat *MAX-TRIES* times
    I := random interpretation
    if $I \models S$ **then return** I
    repeat *MAX-FLIPS* times
      p := an atom such that $flip(I, p)$ satisfies the maximal number of clauses in S
      $I = flip(I, p)$
      if $I \models S$ then return I
  return *don't know*
end

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```
begin
  repeat MAX-TRIES times
    I := random interpretation
    if I ⊨ S then return I
    repeat MAX-FLIPS times
      p := an atom such that flip(I, p) satisfies
                the maximal number of clauses in S
      I = flip(I, p)
      if I ⊨ S then return I
  return don't know
end
```

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      if $I \models S$ **then return** I
  return *don't know*
end

GSAT example

0		0		1
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped atom
	p_1	p_2	p_3		p_1	p_2	p_3		
1	0	0	1	4	3	4	4		
2	0	1	1						
3	0	1	0						
	1	1	0						

GSAT example

0		0		1
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped atom
	p_1	p_2	p_3		p_1	p_2	p_3		
1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4					
3	0	1	0	4					
	1	1	0	5					

GSAT example

0		0		1
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped atom
	p_1	p_2	p_3		p_1	p_2	p_3		
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p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip	interpretation			satisfied clauses				candidates	flipped
no.	p_1	p_2	p_3		p_1	p_2	p_3	for flipping	atom
1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4	3	4	4		
3	0	1	0	4					
	1	1	0	5					

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$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped atom
	p_1	p_2	p_3		p_1	p_2	p_3		
1	0	0	1	4	3	4	4	p_2, p_3	p_2
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$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

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	p_1	p_2	p_3		p_1	p_2	p_3		
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$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

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$\neg p_1$	$\vee$	p_2		
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	p_1	p_2	p_3		p_1	p_2	p_3		
1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4	3	4	4	p_2, p_3	p_3
3	0	1	0	4	5	4	4	p_1	p_1
	1	1	0	5					

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		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip	interpretation			satisfied clauses				candidates	flipped
no.	p_1	p_2	p_3		p_1	p_2	p_3	for flipping	atom
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$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
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	p_1	p_2	p_3		p_1	p_2	p_3		
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$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

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	p_1	p_2	p_3		p_1	p_2	p_3		
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