

Randomised Algorithms for SAT

- ▶ Choose a **random interpretation**.
- ▶ If this interpretation is not a model, repeatedly choose a variable and change its value in the interpretation (**flip** the variable).

The flipped variables are chosen using heuristics or randomly, or both.

$$\text{flip}(I, p)(q) = \begin{cases} I(q), & \text{if } p \neq q; \\ 1, & \text{if } p = q \text{ and } I(p) = 0; \\ 0, & \text{if } p = q \text{ and } I(p) = 1. \end{cases}$$

In other words, the interpretation $\text{flip}(I, p)$ is obtained from I by changing its value on p .

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GSAT

procedure *GSAT*(*S*)

input: set of clauses *S*

output: interpretation *I* such that $I \models S$ or *don't know*

parameters: integers *MAX-TRIES*, *MAX-FLIPS*

begin

repeat *MAX-TRIES* times

I := random interpretation

if $I \models S$ **then return** *I*

$p := \text{random number in } [0, 1]$

$p := \text{number of clauses } C \text{ that } \mathbb{F}_p(I, C) \text{ satisfies}$

$p := \text{number of clauses } C \text{ that } \mathbb{F}_p(I, C) \text{ does not satisfy} \times \text{the number of clauses in } S$

$I = \mathbb{F}_p(I, p)$

if $I \models S$ **then return** *I*

$p := \text{random number in } [0, 1]$

$I = \mathbb{F}_p(I, p)$

end

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repeat *MAX-TRIES* times

I := random interpretation

if $I \models S$ **then return** *I*

repeat *MAX-FLIPS* times

p := an atom such that $\text{flip}(I, p)$ satisfies
the maximal number of clauses in *S*

I = $\text{flip}(I, p)$

if $I \models S$ **then return** *I*

return *don't know*

end

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the maximal number of clauses in *S*

I = *flip*(*I*, *p*)

if $I \models S$ **then return** *I*

return *don't know*

end

GSAT example

	0		0		1
	p_1	∨	$\neg p_2$	∨	p_3
			$\neg p_2$	∨	$\neg p_3$
	$\neg p_1$			∨	$\neg p_3$
	$\neg p_1$	∨	p_2		
	p_1	∨	p_2		

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom
	p_1	p_2	p_3	p_1	p_2	p_3		
1	0	0	1	4	3	4		
2	0	0	0					
3	0	1	1					
4	0	1	0					
5	1	0	1					
6	1	0	0					
7	1	1	1					
8	1	1	0					

GSAT example

	0		0		1
	p_1	∨	$\neg p_2$	∨	p_3
			$\neg p_2$	∨	$\neg p_3$
	$\neg p_1$			∨	$\neg p_3$
	$\neg p_1$	∨	p_2		
	p_1	∨	p_2		

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom
	p_1	p_2	p_3	p_1	p_2	p_3		
1	0	0	1	4	3	4	p_2, p_3	p_2
2	0	1	1	3	4	4	p_1, p_3	p_1
3	0	1	0	3	4	3	p_1, p_2	p_1
	1	1	0	3	4	3	p_1, p_2	p_1

GSAT example

	0		0		1
	p_1	∨	$\neg p_2$	∨	p_3
			$\neg p_2$	∨	$\neg p_3$
	$\neg p_1$			∨	$\neg p_3$
	$\neg p_1$	∨	p_2		
	p_1	∨	p_2		

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom
	p_1	p_2	p_3	p_1	p_2	p_3		
1	0	0	1	4	3	4	p_2, p_3	p_2
2	0	1	1	4				
3	0	1	0	3				
	1	1	0	3				

GSAT example

$$\begin{array}{r}
 \begin{array}{c} 0 \\ \hline p_1 \vee \neg p_2 \\ \neg p_1 \vee p_1 \end{array}
 \quad
 \begin{array}{c} 1 \\ \hline \neg p_2 \vee \neg p_2 \\ \neg p_2 \vee p_2 \end{array}
 \quad
 \begin{array}{c} 1 \\ \hline p_3 \\ \neg p_3 \\ \neg p_3 \end{array}
 \end{array}$$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom
	p_1	p_2	p_3	p_1	p_2	p_3		
1	0	0	1	4	3	4	p_2, p_3	p_2
2	0	1	1	4	3	4	p_2, p_3	p_3
3	0	1	0	3	3	4		
	1	1	0	3	3	4		

GSAT example

$$\begin{array}{r}
 \begin{array}{c} 0 \\ \hline p_1 \vee \neg p_2 \\ \neg p_1 \vee p_2 \\ p_1 \vee p_2 \end{array}
 \quad
 \begin{array}{c} 1 \\ \hline \neg p_2 \vee \neg p_3 \\ \neg p_2 \vee p_3 \\ p_2 \end{array}
 \quad
 \begin{array}{c} 1 \\ \hline p_3 \\ \neg p_3 \end{array}
 \end{array}$$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom
	p_1	p_2	p_3	p_1	p_2	p_3		
1	0	0	1	4	3	4	p_2, p_3	p_2
2	0	1	1	4	3	4	p_2, p_3	p_3
3	0	1	0	4	3	4		
	1	1	0	4	3	4		

GSAT example

$$\begin{array}{r}
 \begin{array}{c} 0 \\ \hline p_1 \vee \neg p_2 \\ \neg p_1 \vee p_2 \\ p_1 \vee p_2 \end{array}
 \quad
 \begin{array}{c} 1 \\ \hline \neg p_2 \vee \neg p_3 \\ \neg p_2 \vee \neg p_3 \\ \vee \neg p_3 \end{array}
 \quad
 \begin{array}{c} 0 \\ \hline p_3 \\ \neg p_3 \end{array}
 \end{array}$$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom
	p_1	p_2	p_3	p_1	p_2	p_3		
1	0	0	1	4	3	4	p_2, p_3	p_2
2	0	1	1	4	3	4	p_2, p_3	p_3
3	0	1	0	4	5	4		
	1	1	0	5	5	4		

GSAT example

$$\begin{array}{r}
 \begin{array}{c} 0 \\ \hline p_1 \vee \neg p_2 \vee p_3 \\ \neg p_1 \vee \neg p_2 \vee \neg p_3 \\ \neg p_1 \vee p_2 \\ p_1 \vee p_2 \end{array}
 \end{array}$$

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped atom
	p_1	p_2	p_3	p_1	p_2	p_3			
1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4	3	4	4	p_2, p_3	p_3
3	0	1	0	4	5	4	4	p_1	p_1
	1	1	0						

GSAT example

$$\begin{array}{r}
 \begin{array}{ccc}
 0 & & 1 & & 0 \\
 \hline
 p_1 & \vee & \neg p_2 & \vee & p_3 \\
 & & \neg p_2 & \vee & \neg p_3 \\
 \neg p_1 & & & \vee & \neg p_3 \\
 \neg p_1 & \vee & p_2 & & \\
 p_1 & \vee & p_2 & &
 \end{array}
 \end{array}$$

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1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4	3	4	4	p_2, p_3	p_3
3	0	1	0	4	5	4	4	p_1	p_1
	1	1	0	5					

GSAT example

$$\begin{array}{r}
 \color{red}{1} \qquad \qquad 1 \qquad \qquad 0 \\
 \hline
 p_1 \vee \neg p_2 \vee p_3 \\
 \qquad \qquad \neg p_2 \vee \neg p_3 \\
 \neg p_1 \qquad \qquad \vee \neg p_3 \\
 \neg p_1 \vee p_2 \\
 p_1 \vee p_2
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3	0	1	0	4	5	4	4	p_1	p_1
	1	1	0	5					

GSAT example

$$\begin{array}{r}
 1 \qquad \qquad 1 \qquad \qquad 0 \\
 \hline
 p_1 \vee \neg p_2 \vee p_3 \\
 \qquad \qquad \neg p_2 \vee \neg p_3 \\
 \neg p_1 \qquad \qquad \vee \neg p_3 \\
 \neg p_1 \vee p_2 \\
 p_1 \vee p_2
 \end{array}$$

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	1	1	0	5					